

2

$$\uparrow N_1 \cos \theta_1 = mg$$

$$\rightarrow N_1 \sin \theta_1 = m(r \sin \theta_1) \omega_1^2$$

$$\uparrow N_2 \cos \theta_2 = mg$$

$$\rightarrow N_2 \sin \theta_2 = m(r \sin \theta_2) \omega_2^2$$

$$\frac{\frac{5}{4}g}{\frac{5}{3}g} = \frac{r\omega_1^2}{r\omega_2^2}$$

$$\frac{\omega_1}{\omega_2} = \frac{1}{2}\sqrt{3}$$

6(a)	Let v be the speed of P before the collision. Energy conservation before the collision: $\frac{1}{2}m(17ag) + \frac{3}{2}mag = \frac{1}{2}mv^2$ $v = \sqrt{20ag}$
6(b)	Let w_P be speed of P after the collision and V be its speed at the top. N2L: $\frac{mV^2}{a} = (T) + mg \quad [V = \sqrt{ag}]$ Energy equation: $\frac{1}{2}mw_P^2 = \frac{1}{2}mV^2 + 2mga$ $w_P = \sqrt{5ag}$
6(c)	Let w_Q the speed of Q after the collision. $mv = kmw_Q - mw_P$ $w_P + w_Q = ev [=v]$ OR $\frac{1}{2}mv^2 = \frac{1}{2}mw_P^2 + \frac{1}{2}kmw_Q^2$ $k = 3$

1	$T_A \cos 45^\circ + T_B \sin 45^\circ = m\omega^2 l \sin 45^\circ$	
	$T_A \sin 45^\circ - T_B \cos 45^\circ = mg$	
	$2T_A = m\omega^2 l + \sqrt{2} mg$	
	$T_A = \frac{1}{2} m(\omega^2 l + \sqrt{2} g)$	
6	A to ground: $\frac{1}{2} mu^2 + 2mga = \frac{1}{2} m(3u)^2$ $\left[u^2 = \frac{1}{2} ga \right]$	
	A to B: $\frac{1}{2} mv^2 + mg a \cos \theta = \frac{1}{2} mu^2 + mga$ $\left[v^2 = u^2 + 2ag(1 - \cos \theta) \right]$	
	B to ground: $\frac{1}{2} mv^2 + mg a \cos \theta = \frac{1}{2} m(3u)^2 - mga$ $\left[v^2 = 9u^2 - 2ag(1 + \cos \theta) \right]$	
	$mg \cos \theta = \frac{mv^2}{a}$ $\left[v^2 = ag \cos \theta \right]$	
	$g \cos \theta = \frac{1}{2} ga + 2ag(1 - \cos \theta)$	
	$\cos \theta = \frac{5}{6}$	
	Horizontally at ground: $v \cos \theta = 3u \cos \beta$.	
	$\cos \beta = \frac{\sqrt{\frac{5}{6} ga} \left(\frac{5}{6} \right)}{3\sqrt{\frac{1}{2} ga}} = \left(\frac{5}{54} \sqrt{15} \right)$	
	OR	
	$v_x = v \cos \theta = \sqrt{\frac{5}{6} ga} \left(\frac{5}{6} \right), \quad v_y^2 = (v \sin \theta)^2 + 2ga(1 + \cos \theta) = \frac{847}{216} ga$	
	$\tan \beta = \frac{\sqrt{\frac{847}{216} ga}}{\sqrt{\frac{5}{6} ga} \left(\frac{5}{6} \right)}$	
	$\beta = 69.0^\circ$	

1	For A : $T = mg$ For B vertically: $T \cos \theta = 0.5mg$
	$\theta = 60^\circ$
	For B horizontally: $T \sin \theta = 0.5m(BR \sin \theta)\omega^2$
	$BR \cos \theta = AR \quad [BR = 8a]$
	$\omega = \frac{1}{2} \sqrt{\frac{g}{a}}$
4(a)	At top $R \geq 0$, so $\frac{mw^2}{a} \geq mg$.
	$\frac{1}{2}mv^2 - \frac{1}{2}mw^2 \geq mga(1 - \cos \theta)$
	$\frac{1}{2}mv^2 \geq \frac{13}{9}mga$
	$v \geq \frac{1}{3}\sqrt{26ag}$
4(b)	Maximum reaction is at bottom: $\frac{1}{2}mv^2 + mga(1 + \cos \theta) = \frac{1}{2}mV^2 \quad [V^2 = 5ag]$
	$R_{\max} - mg = \frac{mV^2}{a}$
	$R_{\max} = 6mg$

3(a)	$8 = \frac{32x}{2}$ leading to $x = 0.5$.
	Hence, $ PA = 2.5$ and $\sin \alpha = \frac{1.5}{2.5} = 0.6$.
3(b)	$(\uparrow) R = 1.6g - 8\cos \alpha$ $(\leftarrow) mr\omega^2 = 8\sin \alpha + \mu R$
	$8\left(\frac{3}{5}\right) + 0.5\left(1.6g - 8\left(\frac{4}{5}\right)\right) = 1.6 \times 1.5\omega^2$
	$\omega^2 = 4$ leading to $\omega = 2$
	Number of revolutions per minute = $\frac{60}{\pi} = 19.1$.

7(a)	$R_A = \frac{mu^2}{a}$ $R_B = \frac{mv^2}{a} - mg \cos \theta$
	$\frac{mu^2}{2a} = \frac{mv^2}{a} - mg \cos \theta \text{ leading to } v^2 = \frac{1}{2}u^2 + agk .$
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + magk \text{ leading to } v^2 = u^2 - 2agk .$
	$\frac{6}{5}ag - 2agk = \frac{3}{5}ag + agk$
	$k = \frac{1}{5}$
7(b)	$\left[R_B = 0 \text{ leading to } u^2 = \frac{3}{5}ag \right] \quad u = \sqrt{\frac{3}{5}ag}$

7(c)

Vertically B to C : $t = \frac{2v \sin \theta}{g}$

Horizontally B to C : $2a \sin \theta = v \cos \theta \times t = v \cos \theta \times \frac{2v \sin \theta}{g}$

$$v^2 = \frac{ag}{\cos \theta}$$

$$u^2 = \frac{ag}{\cos \theta} + 2agk$$

$$u^2 = \frac{27}{5}ag, \quad u = \sqrt{\frac{27}{5}ag}$$

Alternative method for question 7(c)

Equation of trajectory:

$$0 = 2a \sin \theta (\tan \theta) - \frac{g(2a \sin \theta)^2}{2(v \cos \theta)^2} \quad \left[0 = \frac{4}{5}\sqrt{6}a(2\sqrt{6}) - \frac{g\left(\frac{4}{5}\sqrt{6}\right)^2}{\frac{2}{25}v^2} \right]$$

$$v^2 = \frac{ag}{\cos \theta}$$

$$u^2 = \frac{ag}{\cos \theta} + 2agk$$

$$u^2 = \frac{27}{5}ag, \quad u = \sqrt{\frac{27}{5}ag}$$

7(c)

Alternative method for question 7(c)

$$y = x \tan \theta - \frac{gx^2}{2v^2 \cos^2 \theta} \text{ leading to } \frac{dy}{dx} = \tan \theta - \frac{gx}{v^2 \cos^2 \theta}$$

Reaches maximum when $x = a \sin \theta$: $0 = \tan \theta - \frac{ga \sin \theta}{v^2 \cos^2 \theta}$

$$v^2 = \frac{ag}{\cos \theta}$$

$$u^2 = \frac{ag}{\cos \theta} + 2agk$$

$$u^2 = \frac{27}{5} ag, \quad u = \sqrt{\frac{27}{5} ag}$$

1

$$T \cos \theta = mg$$

$$T \sin \theta = m\omega^2 r$$

$$\frac{5}{3} mg \sin \theta = m\omega^2 a \sin \theta$$

$$\omega = \sqrt{\frac{5g}{3a}}$$

5(a)	$\frac{1}{2}mv^2 + mgr \cos \alpha = \frac{1}{2}m\left(\frac{3}{2}gr\right) \quad \left[v^2 = \frac{3}{2}gr - 2gr \cos \alpha\right]$
	$mg \cos \alpha = \frac{mv^2}{r}$
	$mg \cos \alpha = m\left(\frac{3}{2}g - 2g \cos \alpha\right)$
	$\cos \alpha = \frac{1}{2}, \quad \alpha = 60^\circ$
5(b)	$v^2 = \frac{1}{2}gr$
	$0 = \left(\sin 60^\circ \sqrt{v^2}\right)^2 - 2gh$
	$h = \frac{3}{16}r$
	Total height = $\frac{3}{16}r + r \cos 60 = \frac{11}{16}r$