5(a)	$\overline{x} = ka + \frac{1}{3}h$
	$\overline{x} = \frac{1}{3}(3ka + h)$
	$\overline{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$
	Alternative method for question 5(a)
	Coordinates $A(ka,0),\ B(ka+h,h),\ C(ka,2a)$ $\overline{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$
	$\overline{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$

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5(b)	Taking moments about the x-axis: $\overline{x}(2ka^2 + ah) = 2ka^2 \times \frac{1}{2}ka + ah(ka + \frac{1}{3}h)$
	For equilibrium, $\overline{x} \leqslant ka$, so $\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leqslant ka$
	$(0 \leqslant) h \leqslant \sqrt{3}ka$
5(c)	Point of toppling means $h = \sqrt{3}ka\,[= a]$. $\overline{x} = \frac{1}{3}a(\sqrt{3} + 1)$
	[$h = a$, meaning triangle ABC is isosceles.] $\overline{y} = a$ (by symmetry)

3(a)	Vertical distance of COM of sector from OA: $\dfrac{\left(2a\cos\left(\frac{1}{4}\pi\right)\right)\sin\left(\frac{1}{4}\pi\right)}{3\left(\frac{1}{4}\pi\right)}=\dfrac{4a}{3\pi}$
	$ka^2\left(\frac{1}{2}a\right)+\frac{1}{4}\pi a^2\left(a-\dfrac{4a}{3\pi}\right)=\left(ka^2+\frac{1}{4}\pi a^2\right)\bar{y}$ $\left[6ka+3\pi a-4a=3(4k+\pi)\bar{y}\right]$
	$\bar{y}=\dfrac{6k+3\pi-4}{3(4k+\pi)}a$
3(b)	$ka^2\left(\frac{1}{2}ka\right)=\frac{1}{4}\pi a^2\left(\dfrac{4a}{3\pi}\right)$
	$\left[k^2=\frac{2}{3}\right]$ $\quad k=\sqrt{\frac{2}{3}}=0.816$
	Alternative method for question 3(b)
	On the point of toppling, $\bar{x}=ka$: $\dfrac{\left(6k^2+3k\pi+4\right)a}{3(4k+\pi)}=ka$
	$\left[6k^2+4=12k^2\right]$ $\quad k=\sqrt{\frac{2}{3}}=0.816$

3(a)		Area	From AD	From AB
	$ABCD$	$4a^2$	a	a
	AED	a^2	$\frac{1}{3}a$	$\frac{2}{3}a$
	BCF	ah	$2a-\frac{1}{3}h$	$\frac{4}{3}a$
	Shape $BFDE$	$3a^2-ah$	$\bar{x}$	$\bar{y}$
	Taking moments about AD: $\left(3a^2-ah\right)\bar{x}=4a^2\times a-a^2\times\frac{1}{3}a-ah\times\left(2a-\frac{1}{3}h\right)$			
	$\bar{x}=\dfrac{\frac{11}{3}a^3-2a^2h+\frac{1}{3}ah^2}{3a^2-ah}=\dfrac{\frac{1}{3}a(h^2-6ah+11a^2)}{a(3a-h)}=\dfrac{h^2-6ah+11a^2}{3(3a-h)}$			
	Taking moments about AB: $\left(3a^2-ah\right)\bar{y}=4a^2\times a-a^2\times\frac{2}{3}a-\ ah\times\frac{4}{3}a$			
	$\bar{y}=\dfrac{2a(5a-2h)}{3(3a-h)}$			

3(a)	**Alternative method for question 3(a)**
	$\begin{array}{cccc} & \text{Area} & \text{From } AD & \text{From } AB \\ DEF & a(2a-h) & \frac{1}{3}(3a-h) & \frac{4}{3}a \\ EBF & a^2 & \frac{1}{3}(5a-h) & \frac{2}{3}a \\ \text{Shape } BFDE & 3a^2-ah & \bar{x} & \bar{y} \end{array}$
	Taking moments about AD: $\left(3a^2-ah\right)\bar{x}=a\left(2a-h\right)\times\frac{1}{3}(3a-h)+a^2\times\frac{1}{3}(5a-h)$
	$\bar{x}=\dfrac{\frac{1}{3}a(6a^2-5ah+h^2+5a^2-ah)}{3a^2-ah}=\dfrac{\frac{1}{3}a(h^2-6ah+11a^2)}{a(3a-h)}=\dfrac{h^2-6ah+11a^2}{3\left(3a-h\right)}$
	Taking moments about AB: $\left(3a^2-ah\right)\bar{y}=a\left(2a-h\right)\times\frac{4}{3}a+a^2\times\frac{2}{3}a$
	$\bar{y}=\dfrac{2a\left(5a-2h\right)}{3\left(3a-h\right)}$
3(b)	$\bar{x}\geqslant a$ meaning $h^2-3ah+2a^2\geqslant 0$
	$[0<]\, h\leqslant a,\ h=2a$.

4(a)	$\begin{array}{c\|c\|c} & \text{Volume} & \text{CoM above base} \\ \text{Cone} & 18\pi r^3 & \frac{3}{2}r \\ \text{Hemisphere} & \frac{16}{3}\pi r^3 & \frac{3}{4}r \\ \text{Total object} & 18\pi r^3-\frac{16}{3}\pi r^3 & \bar{y} \end{array}$
	$\left(18\pi r^3-\frac{16}{3}\pi r^3\right)\bar{y}=18\pi r^3\left(\frac{3}{2}r\right)-\frac{16}{3}\pi r^3\left(\frac{3}{4}r\right)$
	$\bar{y}=\frac{69}{38}r$
4(b)	When on point of toppling: $\tan\alpha=\dfrac{3r}{their\ \bar{y}}\quad\left[=\frac{38}{23}\right]$
	$\alpha=58.8°$

4

Areas	$2a^2$	$\frac{1}{2}a(2a)$	$2a^2+\frac{1}{2}a(2a)$
Distances from D	$-\frac{1}{2}a$	$\frac{2}{3}a$	$\bar{x}$

$-2a^2\left(\frac{1}{2}a\right)+\frac{1}{2}\left(2a^2\right)\left(\frac{2}{3}a\right)=\left(3a^2\right)\bar{x}$

$m\left(\frac{1}{9}a\right)=km(2a)$

$k=\frac{1}{18}$

Alternative method for question 4

Ratio of masses	$2a^2$	$\frac{1}{2}a(2a)$	$k\left(2a^2+\frac{1}{2}a(2a)\right)$

$2\left(\frac{1}{2}a\right)=1\left(\frac{2}{3}a\right)+3k(2a)$

$k=\frac{1}{18}$