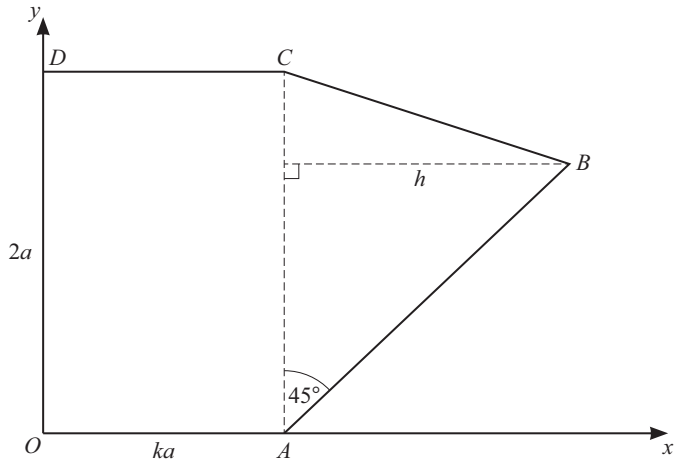
5



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC. The length of OA is ka, the length of OD is $2a$, the height of triangle ABC is h and angle CAB is 45° (see diagram). Relative to axes through O, parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\overline{y}$. [3]

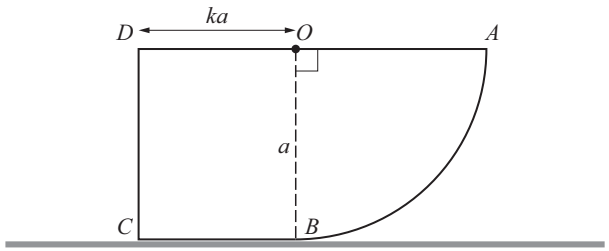
The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

(b) Find, in terms of a and k, the set of values of h for which the lamina is in equilibrium. [4]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

(c) Find, in terms of a, the coordinates of the centre of mass of the triangle ABC. [2]

3 A uniform lamina $OABCD$ is in the form of a rectangle, $OBCD$, joined along the edge OB to a quarter circle OAB. The length of DO is ka and the length of OB is a. The lamina rests in a vertical plane with its edge CB on a horizontal surface (see diagram).



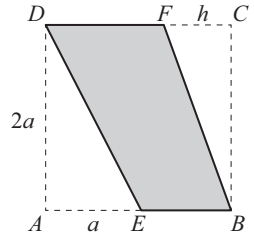
(a) Find, in terms of k, a and π, an expression for the distance of the centre of mass above the horizontal surface. [4]

[You may use without proof the result for the centre of mass of a circular sector in the list of formulae (MF19).]

The lamina is on the point of toppling about B.

(b) Find the value of k. [2]

3



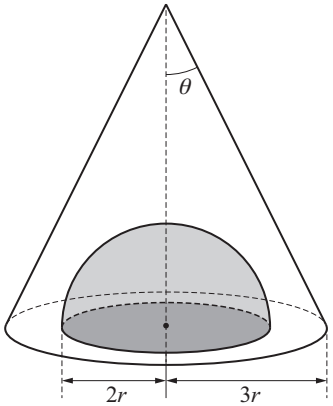
The lamina $BFDE$ is obtained by removing triangles AED and BCF from a uniform square lamina $ABCD$ of side $2a$. The length of side AE is a and the length of side FC is h (see diagram). The centre of mass of $BFDE$ is at a distance $\overline{x}$ from AD, and at a distance $\overline{y}$ from AB.

(a) Show that $\overline{x} = \dfrac{h^2 - 6ah + 11a^2}{3(3a - h)}$ and find a corresponding expression for $\overline{y}$. [5]

(b) The lamina $BFDE$ is placed vertically on its edge EB on a smooth horizontal surface.

Find, in terms of a, the set of possible values of h for which the lamina remains in equilibrium. [2]

4



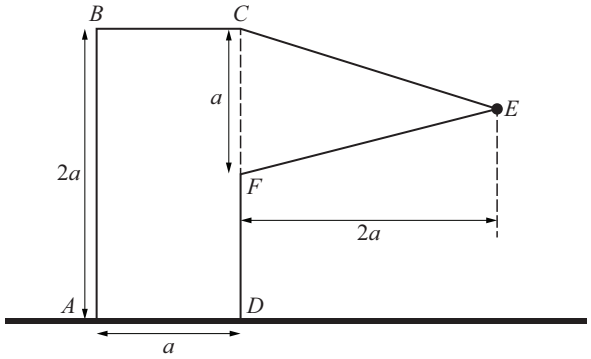
An object is formed by removing a solid hemisphere, radius $2r$, from a uniform solid cone, radius $3r$ and semi-vertical angle θ, where $\tan\theta = \frac{1}{2}$. The axes of symmetry of the cone and the hemisphere coincide. The base of the cone and the base of the hemisphere are in the same plane as each other (see diagram).

(a) Find, in terms of r, the distance of the centre of mass of the object from its base. [4]

The object is placed such that its circular base makes contact with a rough plane which is inclined to the horizontal at an angle α. The object is on the point of toppling. The plane is sufficiently rough to prevent sliding.

(b) Find the value of α. [3]

4



An object consists of a uniform lamina with a particle attached. The uniform lamina $ABCEFD$ of mass m is formed from a rectangle $ABCD$ and an isosceles triangle CEF, where F is the midpoint of CD. The rectangle has sides $AB = 2a$ and $AD = a$. The triangle CEF has base a and height $2a$. The particle of mass km is attached to the lamina at E. The object rests in a vertical plane with its edge AD on horizontal ground (see diagram).

Given that the object is on the point of toppling in its vertical plane about the vertex D, find the value of k. [4]