7(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \tan\theta - \frac{2gx}{2u^2\cos^2\theta}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = -1$
	$-1 = \frac{4}{3} - \frac{2}{45}x_A$
	$x_A = \frac{105}{2}$
	$y_A = \frac{35}{4}$
	Alternative method for question 7(a)
	Horizontally: $v_H = 25\cos\theta = 15$ Vertically: $v_V = 25\sin\theta - gt = 20 - gt$
	At A: $\frac{v_V}{v_H} = -\tan 45° = -1$
	$\frac{20 - gt}{15} = -1,\ t = \frac{7}{2}$
	$x_A = \frac{105}{2}$
	$y_A = \frac{35}{4}$

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7(b)	Let v be the velocity before the collision at A. $u_x = u\cos\theta = 15 = v_x$
	$v_y = -v_x = -15$
	Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2}$ $(\mathrm{m\,s^{-1}})$
7(c)	Let w be the velocity after the collision. $w = ev = \frac{5}{3}\sqrt{2}$
	$-\frac{35}{4} = x\tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2(\cos 45)^2}$, $\frac{9}{5}x^2 - x - \frac{35}{4} = 0$. OR $\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2,\ t = \frac{3}{2}$ $x = (w\cos 45)t\ \left[= \frac{5}{3} \times \frac{3}{2}\right]$
	$x = \frac{5}{2}$ (metres)

4(a)	Horizontal velocities are the same: $U_q\cos\beta = U\cos\alpha$
	$d = (U\sin\alpha)T - \frac{1}{2}gT^2 + (U_q\sin\beta)T + \frac{1}{2}gT^2$
	$d = (U\sin\alpha)T + \left(\dfrac{U\cos\alpha}{\cos\beta}\sin\beta\right)T$
	$d = UT(\sin\alpha + \cos\alpha\tan\beta)$
4(b)	$U\sin\alpha = gT$
	$d = \dfrac{U^2\sin\alpha}{g}(\sin\alpha + \cos\alpha\tan\beta)$
	$d = \dfrac{U^2}{g}$

2(a)	Use trajectory equation from MF19: $y = x\tan\theta - \dfrac{gx^2}{2u^2\cos^2\theta}$. $y = 2x - \dfrac{10x^2}{2u^2 \times \frac{1}{5}} = 2x - \dfrac{25x^2}{u^2}$
2(b)	$12 = 2\times 8 - \dfrac{25\times 8^2}{u^2}$
	$u^2 = 400 \quad [u = 20]$
	$0 = 2D - \dfrac{25D^2}{400}$ leading to $[D = 0,]\ D = 32$.
	$7 = 2D - \dfrac{25D^2}{400}$ leading to $[D = 4,]\ D = 28$.
	$28 \leqslant D \leqslant 32$ only.

2(a)	$10 = 35t - \frac{g}{2\times 24^2}35^2\left(1+t^2\right)$
	$6125t^2 - 20160t + 11885 = 0$
	$\tan\theta = 0.769$ leading to $\theta = 37.6°$.
	$\tan\theta = 2.52$ leading to $\theta = 68.4°$.
2(b)	Use trajectory equation with $y = 0$: $0 = x\tan\theta - \frac{g}{2\times 24^2}x^2(1+\tan^2\theta)$
	39.5
	Alternative method for question 2(b)
	Time of flight, $T = 4.8\sin\theta$ [$= 4.4629\ldots$] $x = 24T\cos\theta$
	39.5
	Alternative method for question 2(b)
	Use range formula: $x = \frac{u^2\sin 2\theta}{g}$
	39.5

7(a)	$v_y = U\sin 45 - gT$ $v_x = U\cos 45$
7(b)	$\frac{v_y}{v_x} = -\tan 60°$
	$\left[\frac{\frac{U}{\sqrt{2}} - gT}{\frac{U}{\sqrt{2}}} = -\sqrt{3}\right]$ $U - \sqrt{2}gT = -\sqrt{3}U$
	$T = \frac{U}{\sqrt{2}g}\left(1+\sqrt{3}\right) = \frac{\sqrt{2}U}{2g}\left(1+\sqrt{3}\right) = \frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)$
7(c)	$D = \frac{U}{\sqrt{2}}\times T$
	$H = \left\lvert\frac{U}{\sqrt{2}}\times T - \tfrac{1}{2}gT^2\right\rvert$
	$\frac{\left\lvert\frac{U}{\sqrt{2}}\times T - \frac{1}{2}gT^2\right\rvert}{\frac{U}{\sqrt{2}}\times T} = \frac{\left\lvert\frac{1}{2}\sqrt{2} - \frac{1}{2}\left(\frac{1}{2}U\left(\sqrt{2}+\sqrt{6}\right)\right)\right\rvert}{\frac{1}{2}\sqrt{2}} = \left\lvert\frac{1-\sqrt{3}}{2}\right\rvert$
	$H : D = 1 : 1+\sqrt{3}$

7(c)	**Alternative method for question 7(c)**
	$D=\frac{U}{\sqrt{2}}\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)$
	$H=\left\lvert\frac{U}{\sqrt{2}}\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)-\frac{1}{2}g\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)^2\right\rvert \quad \left[=\left\lvert-\frac{U^2}{2g}\right\rvert\right]$
	$\frac{\left\lvert\frac{U}{\sqrt{2}}\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)-\frac{1}{2}g\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)^2\right\rvert}{\frac{U}{\sqrt{2}}\left(\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)\right)}=\left\lvert\frac{1-\sqrt{3}}{2}\right\rvert$
	$H:D=1:1+\sqrt{3}$
	Alternative method for question 7(c)
	$D=\frac{U}{\sqrt{2}}\frac{U}{2g}\left(\sqrt{2}+\sqrt{6}\right)$
	$\left(\pm\frac{\sqrt{6}}{2}U\right)^2=\left(\frac{\sqrt{2}}{2}U\right)^2-2gH$
	$(\pm)H=\frac{U^2}{2g}$
	$H:D=1:1+\sqrt{3}$

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7(d)	$w_y=\pm\frac{2}{3}\frac{\sqrt{3}}{\sqrt{2}}U \quad \left[=\pm\frac{\sqrt{2}}{\sqrt{3}}U\right]$
	$w^2=w_y^2+\left(\frac{U}{\sqrt{2}}\right)^2 \quad \left[=\frac{2}{3}U^2+\frac{1}{2}U^2\right]$
	$w=U\sqrt{\frac{2}{3}+\frac{1}{2}}=U\sqrt{\frac{7}{6}}$