

7(a)

$$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$$

$$\frac{dy}{dx} = -1$$

$$-1 = \frac{4}{3} - \frac{2}{45}x_A$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

Alternative method for question 7(a)

Horizontally: $v_H = 25 \cos \theta = 15$

Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$

At A: $\frac{v_V}{v_H} = -\tan 45^\circ = -1$

$$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

7(b)

Let v be the velocity before the collision at A .

$$u_x = u \cos \theta = 15 = v_x$$

$$v_y = -v_x = -15$$

$$\text{Speed: } \sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (ms}^{-1}\text{)}$$

7(c)

Let w be the velocity after the collision.

$$w = ev = \frac{5}{3}\sqrt{2}$$

$$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \quad \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$$

OR

$$\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, \quad t = \frac{3}{2}$$

$$x = (w \cos 45)t \quad \left[= \frac{5}{3} \times \frac{3}{2} \right]$$

$$x = \frac{5}{2} \text{ (metres)}$$

4(a)	Horizontal velocities are the same: $U_q \cos \beta = U \cos \alpha$
	$d = (U \sin \alpha)T - \frac{1}{2}gT^2 + (U_q \sin \beta)T + \frac{1}{2}gT^2$
	$d = (U \sin \alpha)T + \left(\frac{U \cos \alpha}{\cos \beta} \sin \beta \right)T$
	$d = UT(\sin \alpha + \cos \alpha \tan \beta)$
4(b)	$U \sin \alpha = gT$
	$d = \frac{U^2 \sin \alpha}{g}(\sin \alpha + \cos \alpha \tan \beta)$
	$d = \frac{U^2}{g}$

2(a)	Use trajectory equation from MF19: $y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$. $y = 2x - \frac{10x^2}{2u^2 \times \frac{1}{5}} = 2x - \frac{25x^2}{u^2}$
2(b)	$12 = 2 \times 8 - \frac{25 \times 8^2}{u^2}$
	$u^2 = 400 \quad [u = 20]$
	$0 = 2D - \frac{25D^2}{400} \quad \text{leading to} \quad [D = 0,] D = 32.$
	$7 = 2D - \frac{25D^2}{400} \quad \text{leading to} \quad [D = 4,] D = 28.$
	$28 \leq D \leq 32 \text{ only.}$

2(a)	$10 = 35t - \frac{g}{2 \times 24^2} 35^2 (1 + t^2)$
	$6125t^2 - 20160t + 11885 = 0$
	$\tan \theta = 0.769$ leading to $\theta = 37.6^\circ$.
	$\tan \theta = 2.52$ leading to $\theta = 68.4^\circ$.
2(b)	Use trajectory equation with $y = 0$: $0 = x \tan \theta - \frac{g}{2 \times 24^2} x^2 (1 + \tan^2 \theta)$
	39.5
	Alternative method for question 2(b)
	Time of flight, $T = 4.8 \sin \theta$ [= 4.4629...] $x = 24T \cos \theta$
	39.5
	Alternative method for question 2(b)
	Use range formula: $x = \frac{u^2 \sin 2\theta}{g}$
	39.5

7(a)	$v_y = U \sin 45 - gT$ $v_x = U \cos 45$
7(b)	$\frac{v_y}{v_x} = -\tan 60^\circ$ $\left[\frac{\frac{U}{\sqrt{2}} - gT}{\frac{U}{\sqrt{2}}} = -\sqrt{3} \right] \quad U - \sqrt{2}gT = -\sqrt{3}U$ $T = \frac{U}{\sqrt{2}g}(1 + \sqrt{3}) = \frac{\sqrt{2}U}{2g}(1 + \sqrt{3}) = \frac{U}{2g}(\sqrt{2} + \sqrt{6})$
7(c)	$D = \frac{U}{\sqrt{2}} \times T$ $H = \left \frac{U}{\sqrt{2}} \times T - \frac{1}{2}gT^2 \right $ $\frac{\left \frac{U}{\sqrt{2}} \times T - \frac{1}{2}gT^2 \right }{\frac{U}{\sqrt{2}} \times T} = \frac{\left \frac{1}{2}\sqrt{2} - \frac{1}{2}\left(\frac{1}{2}U(\sqrt{2} + \sqrt{6})\right) \right }{\frac{1}{2}\sqrt{2}} = \left \frac{1 - \sqrt{3}}{2} \right $ $H : D = 1 : 1 + \sqrt{3}$

7(c)

Alternative method for question 7(c)

$$D = \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)$$

$$H = \left| \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right) - \frac{1}{2} g \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)^2 \right| \left[= \left| -\frac{U^2}{2g} \right| \right]$$

$$\frac{\left| \frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right) - \frac{1}{2} g \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)^2 \right|}{\frac{U}{\sqrt{2}} \left(\frac{U}{2g} (\sqrt{2} + \sqrt{6}) \right)} = \left| \frac{1 - \sqrt{3}}{2} \right|$$

$$H : D = 1 : 1 + \sqrt{3}$$

Alternative method for question 7(c)

$$D = \frac{U}{\sqrt{2}} \frac{U}{2g} (\sqrt{2} + \sqrt{6})$$

$$\left(\pm \frac{\sqrt{6}}{2} U \right)^2 = \left(\frac{\sqrt{2}}{2} U \right)^2 - 2gH$$

$$(\pm)H = \frac{U^2}{2g}$$

$$H : D = 1 : 1 + \sqrt{3}$$

7(d)

$$w_y = \pm \frac{2}{3} \frac{\sqrt{3}}{\sqrt{2}} U \left[= \pm \frac{\sqrt{2}}{\sqrt{3}} U \right]$$

$$w^2 = w_y^2 + \left(\frac{U}{\sqrt{2}} \right)^2 \left[= \frac{2}{3} U^2 + \frac{1}{2} U^2 \right]$$

$$w = U \sqrt{\frac{2}{3} + \frac{1}{2}} = U \sqrt{\frac{7}{6}}$$