7(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1} x\right) = \tfrac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$
	$= \tfrac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{2}{(1-x)^2} = \frac{1}{1-x^2}$
	Alternative method for question 7(a)
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{\mathrm{d}y}{\mathrm{d}x} = 1$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2}$
7(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x\tanh^{-1} x$
	$\mathrm{e}^{-\int x^{-1}\,\mathrm{d}\theta} = \mathrm{e}^{-\ln x} = x^{-1}$
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-1}y\right) = \tanh^{-1} x$
	$x^{-1}y = x\tanh^{-1} x - \int \frac{x}{1-x^2}\,\mathrm{d}x$
	$x^{-1}y = x\tanh^{-1} x + \tfrac{1}{2}\ln\left(1-x^2\right) + C$
	$0 = \tfrac{1}{2}\tanh^{-1}\tfrac{1}{2} + \tfrac{1}{2}\ln\tfrac{3}{4} + C \Rightarrow C = -\tfrac{1}{4}\ln 3 - \tfrac{1}{2}\ln\tfrac{3}{4}$
	$x^{-1}y = x\tanh^{-1} x + \tfrac{1}{2}\ln\left(1-x^2\right) - \tfrac{1}{4}\ln\tfrac{27}{16}$

5(a)	$m^2+4m+3=0 \Rightarrow m=-1,-3$
	$[y=]Ae^{-x}+Be^{-3x}$
	$y=p\sin x+q\cos x \Rightarrow y'=p\cos x-q\sin x \Rightarrow y''=-p\sin x-q\cos x$
	$-p-4q+3p=0 \quad -q+4p+3q=5$
	$p=1 \quad q=\frac{1}{2}$
	$y=Ae^{-x}+Be^{-3x}+\sin x+\frac{1}{2}\cos x$
5(b)	$y \approx \sin x+\frac{1}{2}\cos x=\frac{\sqrt{5}}{2}\left(\frac{2}{\sqrt{5}}\sin x+\frac{1}{\sqrt{5}}\cos x\right)$
	$\frac{\sqrt{5}}{2}\sin(x+0.464...)$

	$5\theta = \frac{1}{3}\pi + 2k\pi$ or $5\theta = \frac{2}{3}\pi + 2k\pi$
	$\sin\left(\frac{1}{15}\pi\right)$
	$\sin(\frac{2}{15}\pi), \sin(-\frac{5}{15}\pi), \sin(-\frac{4}{15}\pi), \sin(\frac{7}{15}\pi)$
4(a)	$\int_0^1 \frac{1}{2}\left(3^x\right)dx < \frac{1}{2}\left(\left(\frac{1}{N}\right)3^{\frac{1}{N}} + \left(\frac{1}{N}\right)3^{\frac{2}{N}} + \cdots + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}} + \left(\frac{1}{N}\right)3^{\frac{N}{N}}\right)$
	$= \frac{1}{2N}\sum_{n=1}^{N}\left(3^{\frac{1}{N}}\right)^n = \frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}} - 1\right)}$
4(b)	$\int_0^1 \frac{1}{2}\left(3^x\right)dx > \frac{1}{2}\left(\left(\frac{1}{N}\right) + \left(\frac{1}{N}\right)3^{\frac{1}{N}} + \cdots + \left(\frac{1}{N}\right)3^{\frac{N-2}{N}} + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}}\right)$
	$= \frac{1}{2N}\sum_{n=0}^{N-1}\left(3^{\frac{1}{N}}\right)^n = \frac{1}{N\left(3^{\frac{1}{N}} - 1\right)}$
4(c)	$\frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}} - 1\right)} - \frac{1}{N\left(3^{\frac{1}{N}} - 1\right)} = \frac{1}{N}$
	$\frac{1}{N} \to 0$ as $N \to \infty$.

5(a)	$m^2+m+1=0 \Rightarrow m=-\frac{1}{2}\pm\frac{1}{2}\sqrt{3}$
	$y=\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x\right)$
	$y=p\sin 2x+q\cos 2x \Rightarrow y'=2p\cos 2x-2q\sin 2x \Rightarrow y''=-4p\sin 2x-4q\cos 2x$
	$-4p-2q+p=1$ $\qquad$ $-4q+2p+q=1$
	$p=-\frac{1}{13}$ $\qquad$ $q=-\frac{5}{13}$
	$y=\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x\right)-\frac{1}{13}\sin 2x-\frac{5}{13}\cos 2x$
5(b)	$y\approx-\frac{1}{13}\sin 2x-\frac{5}{13}\cos 2x=\sqrt{\frac{2}{13}}\left(-\frac{1}{\sqrt{26}}\sin 2x-\frac{5}{\sqrt{26}}\cos 2x\right)$
	$\sqrt{\frac{2}{13}}\sin(2x-1.77)$

6	$\int \frac{1+x}{1-2x-x^2}\,dx = -\frac{1}{2}\ln\left(1-2x-x^2\right)$
	$e^{-\frac{1}{2}\ln\left(1-2x-x^2\right)} = \frac{1}{\sqrt{1-2x-x^2}}$
	$\frac{d}{dx}\left(\frac{y}{\sqrt{1-2x-x^2}}\right) = \frac{1}{\sqrt{1-2x-x^2}}$
	$1-2x-x^2 = 2-(x+1)^2$
	$\int \frac{1}{\sqrt{2-(x+1)^2}}\,dx = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+C$
	$\pi = \sin^{-1}\frac{1}{\sqrt{2}}+C \Rightarrow C = \frac{3}{4}\pi$
	$\frac{y}{\sqrt{1-2x-x^2}} = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+\frac{3}{4}\pi$

	$5\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}^2 t+1\,dt = 5\left[-\cot t+t\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = 5\left(-\frac{1}{\sqrt{3}}+\frac{1}{3}\pi+\sqrt{3}-\frac{1}{6}\pi\right) = \frac{5}{6}\pi+\frac{10}{3}\sqrt{3}$

7(b)	$\dfrac{\dot{y}}{\dot{x}}=\dfrac{6\operatorname{cosec}t-4\cot^2 t}{8\operatorname{cosec}t+3\cot^2 t}=\dfrac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}=\dfrac{6\sec t-4\cot t}{8\sec t+3\cot t}$
	$\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{6\operatorname{cosec}t-4\cot^2 t}{8\operatorname{cosec}t+3\cot^2 t}\right)$ $=\dfrac{\left(8\csc t+3\cot^2 t\right)\left(-6\cot t\csc t+8\cot t\csc^2 t\right)+\left(6\csc t-4\cot^2 t\right)\left(6\csc^2 t\cot t+8\cot t\csc t\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^2}$ or $\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}\right)$ $=\dfrac{\left(8\sin t+3\cos^2 t\right)\left(6\cos t+8\cos t\sin t\right)-\left(6\sin t-4\cos^2 t\right)\left(8\cos t-6\cos t\sin t\right)}{\left(8\sin t+3\cos^2 t\right)^2}$
	$\dfrac{100\cos t\sin^2 t+50\cos^3 t}{\left(8\sin t+3\cos^2 t\right)^2}=\dfrac{50\cos t\left(\sin^2 t+1\right)}{\left(8\sin t+3\cos^2 t\right)^2}=\dfrac{50\cot t\operatorname{cosec}t\left(\operatorname{cosec}^2 t+1\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^2}$
	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}=\dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}\right)\times\dfrac{\mathrm{d}t}{\mathrm{d}x}=\dfrac{50\sin^2 t\cos t\left(\sin^2 t+1\right)}{\left(8\sin t+3\cos^2 t\right)^3}$ $=\dfrac{50\cot t\operatorname{cosec}t\left(\operatorname{cosec}^2 t+1\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^3}$

8(a)	$\begin{vmatrix} 5 & a & 0 \end{vmatrix}$

6	$\displaystyle\int\frac{1+x}{1-2x-x^2}\,\mathrm{d}x=-\tfrac{1}{2}\ln\left(1-2x-x^2\right)$
	$\mathrm{e}^{-\frac{1}{2}\ln\left(1-2x-x^2\right)}=\dfrac{1}{\sqrt{1-2x-x^2}}$
	$\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{y}{\sqrt{1-2x-x^2}}\right)=\dfrac{1}{\sqrt{1-2x-x^2}}$

5	$6m^2+3m+6=0 \Rightarrow m=-\frac{1}{4}\pm\frac{1}{4}\mathrm{i}\sqrt{15}$
	$x=\mathrm{e}^{-\frac{1}{4}t}\left(A\cos\frac{\sqrt{15}}{4}t+B\sin\frac{\sqrt{15}}{4}t\right)$
	$x=k\mathrm{e}^{-t} \Rightarrow x'=-k\mathrm{e}^{-t} \Rightarrow x''=k\mathrm{e}^{-t}$
	$6k\mathrm{e}^{-t}-3k\mathrm{e}^{-t}+6k\mathrm{e}^{-t}=\mathrm{e}^{-t}$
	$k=\frac{1}{9}$
	$x=\mathrm{e}^{-\frac{1}{4}t}\left(A\cos\frac{\sqrt{15}}{4}t+B\sin\frac{\sqrt{15}}{4}t\right)+\frac{1}{9}\mathrm{e}^{-t}$
	$x'=\frac{\sqrt{15}}{4}\mathrm{e}^{-\frac{1}{4}t}\left(-A\sin\frac{\sqrt{15}}{4}t+B\cos\frac{\sqrt{15}}{4}t\right)-\frac{1}{4}\mathrm{e}^{-\frac{1}{4}t}\left(A\cos\frac{\sqrt{15}}{4}t+B\sin\frac{\sqrt{15}}{4}t\right)-\frac{1}{9}\mathrm{e}^{-t}$
	$0=A+\frac{1}{9} \quad 0=\frac{\sqrt{15}}{4}B-\frac{1}{4}A-\frac{1}{9} \Rightarrow A=-\frac{1}{9}, \quad B=\frac{1}{3\sqrt{15}}$
	$x=\frac{1}{3}\mathrm{e}^{-\frac{1}{4}t}\left(-\frac{1}{3}\cos\frac{\sqrt{15}}{4}t+\frac{1}{\sqrt{15}}\sin\frac{\sqrt{15}}{4}t\right)+\frac{1}{9}\mathrm{e}^{-t}$

7	$\int \frac{x+5}{x^2+10x+61}\,dx = \frac{1}{2}\ln\left(x^2+10x+61\right)$
	$e^{-\frac{1}{2}\ln\left(x^2+10x+61\right)} = \frac{1}{\sqrt{x^2+10x+61}}$
	$\frac{d}{dx}\left(\frac{y}{\sqrt{x^2+10x+61}}\right) = \frac{1}{\sqrt{x^2+10x+61}}$
	$x^2+10x+61 = (x+5)^2+36$
	$\int \frac{1}{\sqrt{(x+5)^2+6^2}}\,dx = \sinh^{-1}\left(\frac{x+5}{6}\right)+C$
	$C = -\sinh^{-1}\left(\frac{8}{6}\right) = -\ln\left(\frac{8}{6}+\frac{5}{3}\right) = -\ln 3$
	$\frac{y}{\sqrt{x^2+10x+61}} = \sinh^{-1}\left(\frac{x+5}{6}\right) - \sinh^{-1}\left(\frac{4}{3}\right)$

4	$m^2+m-2=0 \Rightarrow m=1,-2$
	$x=Ae^t+Be^{-2t}$
	$x=pt^2+qt+r \Rightarrow x'=2pt+q \Rightarrow x''=2p$
	$p=-1 \qquad 2p-2q=1 \qquad 2p+q-2r=-1$
	$p=-1 \qquad q=-\frac{3}{2} \qquad r=-\frac{5}{4}$
	$x=Ae^t+Be^{-2t}-t^2-\frac{3}{2}t-\frac{5}{4}$
	$x'=Ae^t-2Be^{-2t}-2t-\frac{3}{2}$
	$A+B-\frac{5}{4}=0 \quad A-2B-\frac{3}{2}=0 \Rightarrow A=\frac{4}{3},\ B=-\frac{1}{12}$
	$x=\frac{4}{3}e^t-\frac{1}{12}e^{-2t}-t^2-\frac{3}{2}t-\frac{5}{4}$

7	$\int \frac{2x+6}{x^2+6x+5}\,dx = \ln\left(x^2+6x+5\right)$
	$e^{-\ln\left(x^2+6x+5\right)} = \frac{1}{x^2+6x+5}$
	$\frac{d}{dx}\left(\frac{y}{x^2+6x+5}\right) = \frac{4}{x^2+6x+5}$
	$\frac{4}{x^2+6x+5} = \frac{1}{x+1} - \frac{1}{x+5}$
	$\int \frac{4}{x^2+6x+5}\,dx = \ln\left(\frac{x+1}{x+5}\right) + C$
	$0 = \ln\frac{1}{5} + C$
	$\frac{y}{x^2+6x+5} = \ln\left(\frac{x+1}{x+5}\right) - \ln\frac{1}{5} = \ln\left(\frac{5x+5}{x+5}\right)$