4 Find the particular solution of the differential equation

$$5\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x^2+5x+3,$$

given that, when $x=0$, $y=\frac{dy}{dx}=0$. [10]

7 **(a)** Show that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1}x\right) = \frac{1}{1-x^2}$. [3]

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(b) Find the solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} - y = x^2 \tanh^{-1}x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

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5 **(a)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 5\cos x.$$ [7]

(b) For large positive values of x and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R\sin(x+\phi),$$

where the constants R and ϕ are to be determined. [3]

5 **(a)** Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x + \cos 2x. \qquad [7]$$

(b) For large positive values of x and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R\sin(2x-\phi),$$

where R and ϕ are positive constants to be determined. [3]

6 Find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{1+x}{1-2x-x^2}y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form. [10]

5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2}+3\frac{dx}{dt}+6x=e^{-t},$$

given that, when $t=0$, $x=\frac{dx}{dt}=0$. [10]

7 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form. [10]

4 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 2t^2 + t - 1,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$. [10]

7 Find the solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2x+6}{x^2+6x+5}y = 4,$$

given that $y = 0$ when $x = 0$. Give your answer in an exact form. [9]