

4

$$5\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x^2 + 5x + 3,$$

given that, when $x = 0$, $y = \frac{dy}{dx} = 0$.

[10]

[illegible]

7 (a) Show that $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$.

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

(b) For large positive values of x and for any initial conditions, show that the solution to part (a) can be approximated by

$$y \approx R \sin(x + \phi),$$

where the constants R and ϕ are to be determined. [3]

- (b)** For large positive values of x and for any initial conditions, show that the solution to part **(a)** can be approximated by

$$y \approx R \sin(2x - \phi),$$

where R and ϕ are positive constants to be determined.

[3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 Find the particular solution of the differential equation

$$\frac{dy}{dx} + \frac{1+x}{1-2x-x^2} y = 1$$

given that $y = \pi$ when $x = 0$. Give your answer in an exact form. [10]

[illegible]

5 Find the particular solution of the differential equation

$$6\frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 6x = e^{-t},$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{x+5}{x^2+10x+61}y = 1,$$

given that $y = 0$ when $x = 3$. Give your answer in an exact form.

[10]

Handwriting practice lines on page 13.

4 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} - 2x = 2t^2 + t - 1,$$

given that, when $t = 0$, $x = \frac{dx}{dt} = 0$.

[10]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 Find the solution of the differential equation

$$\frac{dy}{dx} - \frac{2x+6}{x^2+6x+5}y = 4,$$

given that $y = 0$ when $x = 0$. Give your answer in an exact form.

[9]

Blank lined paper for writing.

