8 **(a)** State the sum of the series $z+z^2+...+z^n$, for $z \neq 1$. [1]

(b) By letting $z=\frac{1}{2}(\cos\theta+\mathrm{i}\sin\theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^{n}\left(\frac{1}{2}\right)^m \sin m\theta=\frac{\left(\frac{1}{2}\right)^{n+2}\sin n\theta-\left(\frac{1}{2}\right)^{n+1}\sin(n+1)\theta+\frac{1}{2}\sin\theta}{\frac{5}{4}-\cos\theta}.$$ [6]

(c) Use the result in **(b)** to find $\sum_{m=1}^{n}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of n and θ. [You do not need to simplify your answer.] [3]

(d) Hence find $\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of $\cos\theta$. [You may assume that $\frac{n}{2^n} \to 0$ as $n \to \infty$.] [2]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

2 Find the roots of the equation $z^4 = 8 - 8\mathrm{i}\sqrt{3}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

4 **(a)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^6\theta = a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d,$$

where a, b, c and d are constants to be determined. [5]

(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 2x \, \mathrm{d}x$. [3]

3 **(a)** Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta. \qquad [4]$$

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(b) Hence obtain the roots of the equation

$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where q is a rational number. [4]

1 Find the roots of the equation $z^3 = 27 - 27\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi \leqslant \theta < \pi$. [5]

3 By considering the binomial expansion of $\left(z - \frac{1}{z}\right)$, where $z = \cos\theta + \mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\operatorname{cosec}^5\theta = \frac{a}{\sin 5\theta + b\sin 3\theta + c\sin\theta},$$

where a, b and c are integers to be determined. [6]

5 **(a)** Use de Moivre's theorem to show that

$$\sec 5\theta = \frac{\sec^5\theta}{5\sec^4\theta - 20\sec^2\theta + 16}.$$ [6]

(b) Hence, obtain the roots of the equation

$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\sec(q\pi)$, where q is rational. [4]