

8 (a) State the sum of the series $z + z^2 + \dots + z^n$, for $z \neq 1$.

[1]

(b) By letting $z = \frac{1}{2}(\cos \theta + i \sin \theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^n \left(\frac{1}{2}\right)^m \sin m\theta = \frac{\left(\frac{1}{2}\right)^{n+2} \sin n\theta - \left(\frac{1}{2}\right)^{n+1} \sin(n+1)\theta + \frac{1}{2} \sin \theta}{\frac{5}{4} - \cos \theta}. \quad [6]$$

[6]

(c) Use the result in (b) to find $\sum_{m=1}^n \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of n and θ . [You do not need to simplify your answer.] [3]

(d) Hence find $\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m m \cos m\theta$ in terms of $\cos \theta$. [You may assume that $\frac{n}{2^n} \rightarrow 0$ as $n \rightarrow \infty$.] [2]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 4 (a) By considering the binomial expansion of $\left(z + \frac{1}{z}\right)$, where $z = \cos \theta + i \sin \theta$, use de Moivre's theorem to show that

$$\cos^6 \theta = a \cos 6\theta + b \cos 4\theta + c \cos 2\theta + d,$$

where a, b, c and d are constants to be determined.

[5]

[illegible]

3 (a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta. \quad [4]$$

Handwriting practice lines (dotted lines) for the letter 'z'.

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(b)

$$32x^5 - 40x^3 + 10x - \sqrt{3} = 0$$

in the form $\sin q\pi$, where q is a rational number. [4]

[4]

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$$\operatorname{cosec}^5 \theta = \frac{a}{\sin 5\theta + b \sin 3\theta + c \sin \theta},$$

where a, b and c are integers to be determined.

[6]

[illegible]

5 (a) Use de Moivre's theorem to show that

$$\sec 5\theta = \frac{\sec^5 \theta}{5 \sec^4 \theta - 20 \sec^2 \theta + 16}. \quad [6]$$

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(b)

$$\sqrt{3}x^5 - 10x^4 + 40x^2 - 32 = 0$$

in the form $\sec(q\pi)$, where q is rational.

[4]

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