

3(a)	$\frac{d}{dx} \left(x(1+x^2)^n \right) = 2nx^2(1+x^2)^{n-1} + (1+x^2)^n$
	$= 2n(1+x^2-1)(1+x^2)^{n-1} + (1+x^2)^n$
	$\left[x(1+x^2)^n \right]_0^1 = 2nI_n - 2nI_{n-1} + I_n$
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$
3(b)	$I_{-1} = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4}\pi$
	$-I_{-1} = \frac{1}{2} - 2I_{-2}$
	$-\frac{1}{4}\pi = \frac{1}{2} - 2I_{-2} \Rightarrow I_{-2} = \frac{1}{4} + \frac{1}{8}\pi$

4	$5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}i$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right)$
	$y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$
	$p=1 \quad 4p+q=5 \quad 10p+2q+r=3$
	$q=1 \quad r=-9$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + x^2 + x - 9$
	$y' = e^{-\frac{1}{5}x} \left(-\frac{2}{5}A \sin \frac{2}{5}x + \frac{2}{5}B \cos \frac{2}{5}x \right) - \frac{1}{5}e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + 2x + 1$
	$A-9=0 \quad \frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A=9, B=2$
	$y = e^{-\frac{1}{5}x} \left(9 \cos \frac{2}{5}x + 2 \sin \frac{2}{5}x \right) + x^2 + x - 9$

$$\frac{4}{3n} \rightarrow 0 \text{ as } n \rightarrow \infty$$

4	$5m^2 + 2m + 1 = 0 \Rightarrow m = -\frac{1}{5} \pm \frac{2}{5}i$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right)$
	$y = px^2 + qx + r \Rightarrow y' = 2px + q \Rightarrow y'' = 2p$
	$p = 1 \quad 4p + q = 5 \quad 10p + 2q + r = 3$
	$q = 1 \quad r = -9$
	$y = e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + x^2 + x - 9$
	$y' = e^{-\frac{1}{5}x} \left(-\frac{2}{5}A \sin \frac{2}{5}x + \frac{2}{5}B \cos \frac{2}{5}x \right) - \frac{1}{5}e^{-\frac{1}{5}x} \left(A \cos \frac{2}{5}x + B \sin \frac{2}{5}x \right) + 2x + 1$
	$A - 9 = 0 \quad \frac{2}{5}B - \frac{1}{5}A + 1 = 0 \Rightarrow A = 9, B = 2$
	$y = e^{-\frac{1}{5}x} \left(9 \cos \frac{2}{5}x + 2 \sin \frac{2}{5}x \right) + x^2 + x - 9$

5(a)	$\left[\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx < \right] \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n}{n} \right)^3 + \left(\frac{n}{n} \right) \right)$
	$= \frac{1}{3n^4} \sum_{r=1}^n r^3 + \frac{1}{n^2} \sum_{r=1}^n r = \frac{n^2(n+1)^2}{12n^4} + \frac{n(n+1)}{2n^2}$
	$= \frac{1}{12} \left(1 + \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 + \frac{1}{n} \right) = \frac{1}{12} \left(1 + \frac{1}{n} \right) \left(7 + \frac{1}{n} \right)$
5(b)	$\int_0^1 \left(\frac{1}{3}x^3 + x \right) dx \geq \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{1}{n} \right)^3 + \left(\frac{1}{n} \right) \right) + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{2}{n} \right)^3 + \left(\frac{2}{n} \right) \right) + \dots + \left(\frac{1}{n} \right) \left(\frac{1}{3} \left(\frac{n-1}{n} \right)^3 + \left(\frac{n-1}{n} \right) \right)$
	$= \frac{1}{3n^4} \sum_{r=1}^{n-1} r^3 + \frac{1}{n^2} \sum_{r=1}^{n-1} r = \frac{(n-1)^2 n^2}{12n^4} + \frac{(n-1)n}{2n^2}$
	$= \frac{1}{12} \left(1 - \frac{1}{n} \right)^2 + \frac{1}{2} \left(1 - \frac{1}{n} \right) = \frac{1}{12} \left(1 - \frac{1}{n} \right) \left(7 - \frac{1}{n} \right)$
5(c)	$U_n - L_n = \frac{1}{12} \left(\frac{16}{n} \right) = \frac{4}{3n}$
	$\frac{4}{3n} \rightarrow 0 \text{ as } n \rightarrow \infty$

1	$s = \int_0^\pi \sqrt{e^{\frac{8}{3}\theta} + \frac{16}{9} e^{\frac{8}{3}\theta}} d\theta = \frac{5}{3} \int_0^\pi e^{\frac{4}{3}\theta} d\theta$
	$s = \frac{5}{4} \left[e^{\frac{4}{3}\theta} \right]_0^\pi$
	$= \frac{5}{4} \left(e^{\frac{4}{3}\pi} - 1 \right)$

6(a)	$\frac{dx}{du} = \frac{1}{2}\sqrt{2} \cosh u \Rightarrow \int \frac{1}{\sqrt{2x^2+1}} dx = \frac{1}{2}\sqrt{2} \int \frac{\cosh u}{\sqrt{\sinh^2 u + 1}} du = \frac{1}{2}\sqrt{2} \int 1 du$
	$= \frac{1}{2}\sqrt{2} \sinh^{-1}(x\sqrt{2}) [+ C]$
6(b)	$\frac{1}{n} \left(\frac{1}{\sqrt{2 \times \frac{1^2}{n^2} + 1}} + \frac{1}{\sqrt{2 \times \frac{2^2}{n^2} + 1}} + \dots + \frac{1}{\sqrt{2 \times \frac{n^2}{n^2} + 1}} \right)$
	$= \frac{1}{n} \left(\frac{n}{\sqrt{2 \times 1^2 + n^2}} + \frac{n}{\sqrt{2 \times 2^2 + n^2}} + \dots + \frac{n}{\sqrt{2 \times n^2 + n^2}} \right)$
	$< \int_0^1 \frac{1}{\sqrt{2x^2+1}} dx$
	$\int_0^1 \frac{1}{\sqrt{2x^2+1}} dx = \left[\frac{1}{2}\sqrt{2} \sinh^{-1}(x\sqrt{2}) \right]_0^1 = \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$
	$\frac{1}{n} \sum_{r=1}^n \frac{n}{\sqrt{2r^2+n^2}} = \sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$

6(c)	$\frac{1}{n} \left(\frac{1}{\sqrt{2 \times 0^2 + 1}} + \frac{1}{\sqrt{2 \times \frac{1^2}{n^2} + 1}} + \dots + \frac{1}{\sqrt{2 \times \frac{(n-1)^2}{n^2} + 1}} \right)$
	$= \frac{1}{n} \left(1 + \frac{n}{\sqrt{2 \times 1^2 + n^2}} + \dots + \frac{n}{\sqrt{2(n-1)^2 + n^2}} \right)$
	$\geq \int_0^1 \frac{1}{\sqrt{2x^2+1}} dx = \left[\frac{1}{2}\sqrt{2} \sinh^{-1}(x\sqrt{2}) \right]_0^1 = \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$
	$\frac{1}{n} + \sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} - \frac{1}{\sqrt{3n^2}} \geq \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$
6(d)	$\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} \geq \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3}) + \frac{1}{n} \left(\frac{1}{3}\sqrt{3} - 1 \right)$
6(d)	$\frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3})$

1(a)	$I_n = \left[x^n \cosh x \right]_0^1 - n \int_0^1 x^{n-1} \cosh x dx$
	$= \left[x^n \cosh x \right]_0^1 - n \left(\left[x^{n-1} \sinh x \right]_0^1 - (n-1) \int_0^1 x^{n-2} \sinh x dx \right)$
	$I_n = \cosh 1 - n \left(\sinh 1 - (n-1) I_{n-2} \right) = \cosh 1 - n \sinh 1 + n(n-1) I_{n-2}$
1(b)	$I_2 = \cosh 1 - 2 \sinh 1 + 2 I_0$
	$I_2 = 3 \cosh 1 - 2 \sinh 1 - 2$

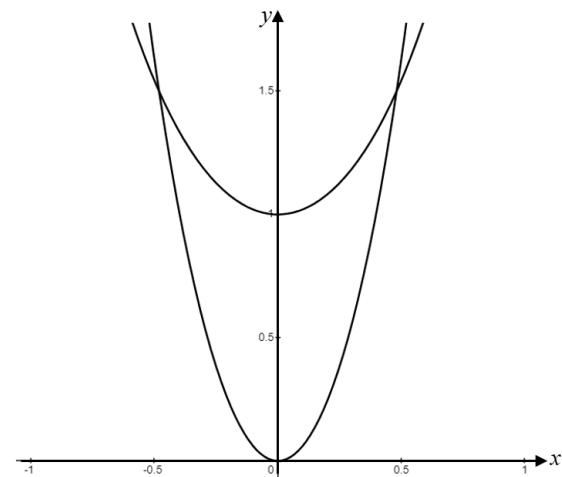
2(a)

$$1 + 2 \sinh^2 x = 6 \sinh^2 x \Rightarrow \sinh x = \pm \frac{1}{2}$$

$$x = \sinh^{-1} \left(\pm \frac{1}{2} \right) = \ln \left(\pm \frac{1}{2} + \sqrt{\frac{1}{4} + 1} \right)$$

$$x = \ln \left(\pm \frac{1}{2} + \frac{1}{2} \sqrt{5} \right)$$

2(b)



3(a)	$\sin 5\theta = \operatorname{Im}(\cos \theta + i \sin \theta)^5 = \sin^5 \theta - 10 \sin^3 \theta \cos^2 \theta + 5 \sin \theta \cos^4 \theta$
	$= \sin^5 \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + 5 \sin \theta (1 - \sin^2 \theta)^2$
	$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$
3(b)	$x = \sin \theta, \quad \sin 5\theta = \frac{1}{2} \sqrt{3}$
	$5\theta = \frac{1}{3} \pi + 2k\pi \quad \text{or} \quad 5\theta = \frac{2}{3} \pi + 2k\pi$
	$\sin\left(\frac{1}{15} \pi\right)$
	$\sin\left(\frac{2}{15} \pi\right), \sin\left(-\frac{5}{15} \pi\right), \sin\left(-\frac{4}{15} \pi\right), \sin\left(\frac{7}{15} \pi\right)$
	$\frac{1}{N} \rightarrow 0 \text{ as } N \rightarrow \infty.$

5(a)	$m^2 + m + 1 = 0 \Rightarrow m = -\frac{1}{2} \pm \frac{1}{2} \sqrt{3}$
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2} x + B \sin \frac{\sqrt{3}}{2} x \right)$
	$y = p \sin 2x + q \cos 2x \Rightarrow y' = 2p \cos 2x - 2q \sin 2x \Rightarrow y'' = -4p \sin 2x - 4q \cos 2x$
	$-4p - 2q + p = 1 \qquad -4q + 2p + q = 1$
	$p = -\frac{1}{13} \qquad q = -\frac{5}{13}$
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2} x + B \sin \frac{\sqrt{3}}{2} x \right) - \frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x$
5(b)	$y \approx -\frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x = \sqrt{\frac{2}{13}} \left(-\frac{1}{\sqrt{26}} \sin 2x - \frac{5}{\sqrt{26}} \cos 2x \right)$
	$\sqrt{\frac{2}{13}} \sin(2x - 1.77)$

$$1 - 2x - x^2 = 2 - (x+1)^2$$

$$\int \frac{1}{\sqrt{2 - (x+1)^2}} dx = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + C$$

$$\pi = \sin^{-1} \frac{1}{\sqrt{2}} + C \Rightarrow C = \frac{3}{4} \pi$$

$$\frac{y}{\sqrt{1 - 2x - x^2}} = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + \frac{3}{4} \pi$$

4(a)	$\int_0^1 \frac{1}{2} (3^x) dx < \frac{1}{2} \left(\left(\frac{1}{N} \right) 3^{\frac{1}{N}} + \left(\frac{1}{N} \right) 3^{\frac{2}{N}} + \dots + \left(\frac{1}{N} \right) 3^{\frac{N-1}{N}} + \left(\frac{1}{N} \right) 3^{\frac{N}{N}} \right)$
	$= \frac{1}{2N} \sum_{n=1}^N \left(3^{\frac{n}{N}} \right)^n = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}$
4(b)	$\int_0^1 \frac{1}{2} (3^x) dx > \frac{1}{2} \left(\left(\frac{1}{N} \right) + \left(\frac{1}{N} \right) 3^{\frac{1}{N}} + \dots + \left(\frac{1}{N} \right) 3^{\frac{N-2}{N}} + \left(\frac{1}{N} \right) 3^{\frac{N-1}{N}} \right)$
	$= \frac{1}{2N} \sum_{n=0}^{N-1} \left(3^{\frac{n}{N}} \right)^n = \frac{1}{N(3^{\frac{1}{N}} - 1)}$
4(c)	$\frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)} - \frac{1}{N(3^{\frac{1}{N}} - 1)} = \frac{1}{N}$

2(a)	$I_n = \int_0^1 (1-x)^n \sinh x dx = \left[(1-x)^n \cosh x \right]_0^1 + \int_0^1 n(1-x)^{n-1} \cosh x dx$
	$I_n = -1 + n \left[(1-x)^{n-1} \sinh x \right]_0^1 + n(n-1) \int_0^1 (1-x)^{n-2} \sinh x dx$
	$I_n = -1 + n(n-1)I_{n-2}$
2(b)	$I_0 = \int_0^1 \sinh x dx = \left[\cosh x \right]_0^1 = \cosh 1 - 1.$
	$I_2 = -1 + 2I_0$
	$I_2 = 2 \cosh 1 - 3$

4(a)	$\frac{1}{\sqrt{1}} e^{\sqrt{1}} + \frac{1}{\sqrt{2}} e^{\sqrt{2}} + \dots + \frac{1}{\sqrt{n-1}} e^{\sqrt{n-1}}$
	$< \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx$
	$\int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left[2e^{\sqrt{x}} \right]_1^n + \frac{1}{\sqrt{n}} e^{\sqrt{n}} = \left(2 + \frac{1}{\sqrt{n}} \right) e^{\sqrt{n}} - 2e$
4(b)	$\frac{1}{\sqrt{2}} e^{\sqrt{2}} + \frac{1}{\sqrt{3}} e^{\sqrt{3}} + \dots + \frac{1}{\sqrt{n}} e^{\sqrt{n}}$
	$> \int_1^n \frac{1}{\sqrt{x}} e^{\sqrt{x}} dx = \left[2e^{\sqrt{x}} \right]_1^n$
	$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} > \left[2e^{\sqrt{x}} \right]_1^n + e = 2e^{\sqrt{n}} - e$

6(a)	$\frac{1}{n} \left(\frac{1}{\frac{1^2}{n^2} + 1} + \frac{1}{\frac{2^2}{n^2} + 1} + \dots + \frac{1}{\frac{n^2}{n^2} + 1} \right) = \frac{1}{n} \left(\frac{n^2}{1^2 + n^2} + \frac{n^2}{2^2 + n^2} + \dots + \frac{n^2}{n^2 + n^2} \right)$
	$\int_0^1 \frac{1}{x^2 + 1} dx = \left[\tan^{-1} x \right]_0^1 = \frac{1}{4} \pi$
	$\frac{1}{n} \sum_{r=1}^n \frac{n^2}{n^2 + r^2} = \sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4} \pi$
6(b)	$\frac{1}{n} \left(\frac{1}{0^2 + 1} + \frac{1}{\frac{1^2}{n^2} + 1} + \dots + \frac{1}{\frac{(n-1)^2}{n^2} + 1} \right) = \frac{1}{n} \left(\frac{n^2}{0^2 + n^2} + \frac{n^2}{1^2 + n^2} + \dots + \frac{n^2}{(n-1)^2 + n^2} \right)$
	$\frac{1}{n} + \sum_{r=1}^n \frac{n}{n^2 + r^2} - \frac{1}{2n} \geq \frac{1}{4} \pi \Rightarrow \sum_{r=1}^n \frac{n}{n^2 + r^2} \geq \frac{1}{4} \pi - \frac{1}{2n}$
6(c)	$\frac{1}{4} \pi$