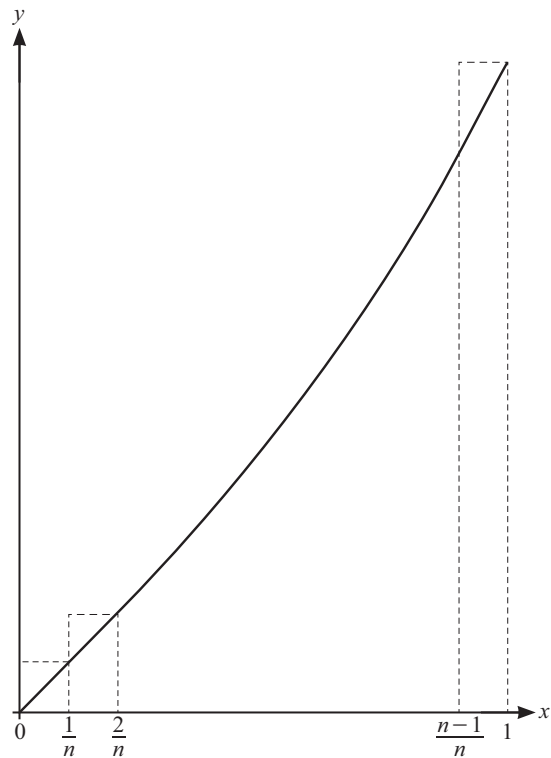


3 The integral I_n is defined by $I_n = \int_0^1 (1+x^2)^n \, dx$.

(a) By considering $\frac{d}{dx}(x(1+x^2)^n)$, or otherwise, show that

$$(2n+1)I_n = 2^n + 2nI_{n-1}.$$
 [5]

(b) Find the exact value of I_{-2} . [4]



The diagram shows the curve with equation $y = \frac{1}{3}x^3 + x$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx < U_n$, where

$$U_n = \frac{1}{12} \left(1 + \frac{1}{n}\right) \left(7 + \frac{1}{n}\right).$$
 [5]

(b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{1}{3}x^3 + x\right) dx$. [4]

(c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]

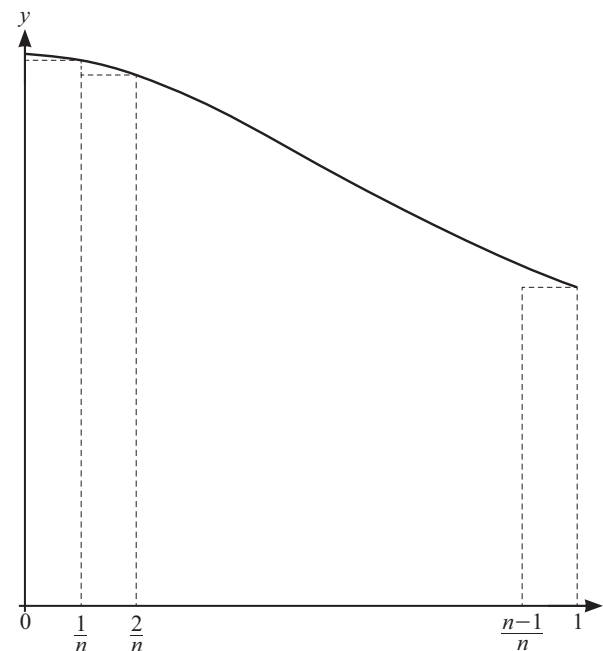
- 1 The curve C has polar equation $r = e^{\frac{\pi}{3}\theta}$ for $0 \leq \theta \leq \pi$.

Find the length of C . Give your answer in an exact form.

[4]

- 6 (a) Use the substitution $x = \frac{1}{2}\sqrt{2} \sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}} dx$.

[3]



The diagram shows the curve with equation $y = \frac{1}{\sqrt{2x^2+1}}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{2r^2+n^2}} < \frac{1}{2}\sqrt{2} \ln(\sqrt{2} + \sqrt{3}).$$

[5]

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(c) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [4]

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(d) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{1}{\sqrt{2r^2 + n^2}}$. [1]

1 It is given that $I_n = \int_0^1 x^n \sinh x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$,

$I_n = \cosh 1 - n \sinh 1 + n(n-1)I_{n-2}.$ [3]

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(b) Find the exact value of I_2 . [2]

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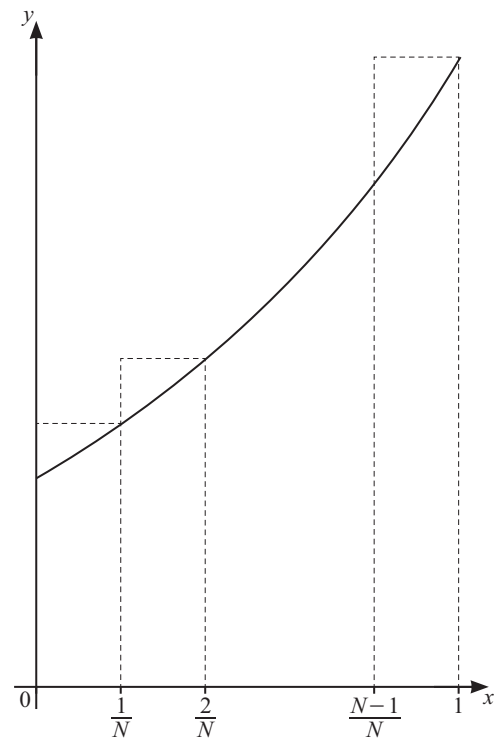
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The diagram shows the curve with equation $y = \frac{1}{2}(3^x)$ for $0 \leq x \leq 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}(3^x) dx < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N(3^{\frac{1}{N}} - 1)}.$$
 [4]

(b) Use a similar method to find, in terms of N , a lower bound L_N for $\int_0^1 \frac{1}{2}(3^x) dx$. [4]

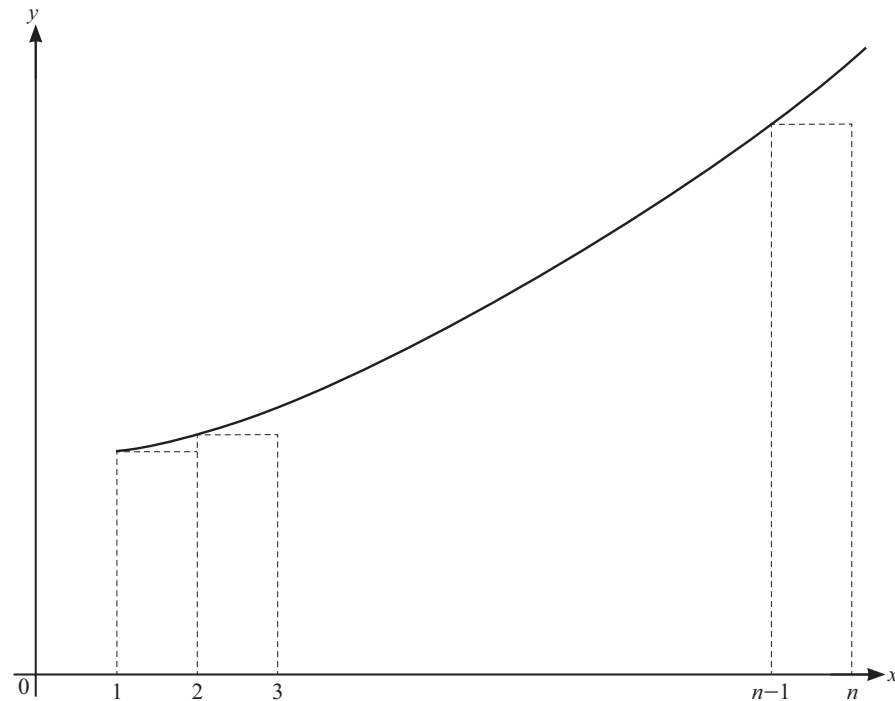
(c) By simplifying $U_N - L_N$, show that $\lim_{N \rightarrow \infty} (U_N - L_N) = 0$. [2]

2 Let $I_n = \int_0^1 (1-x)^n \sinh x \, dx$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$, $I_n = -1 + n(n-1)I_{n-2}$. [4]

(b) Find the exact value of I_2 . [3]

4



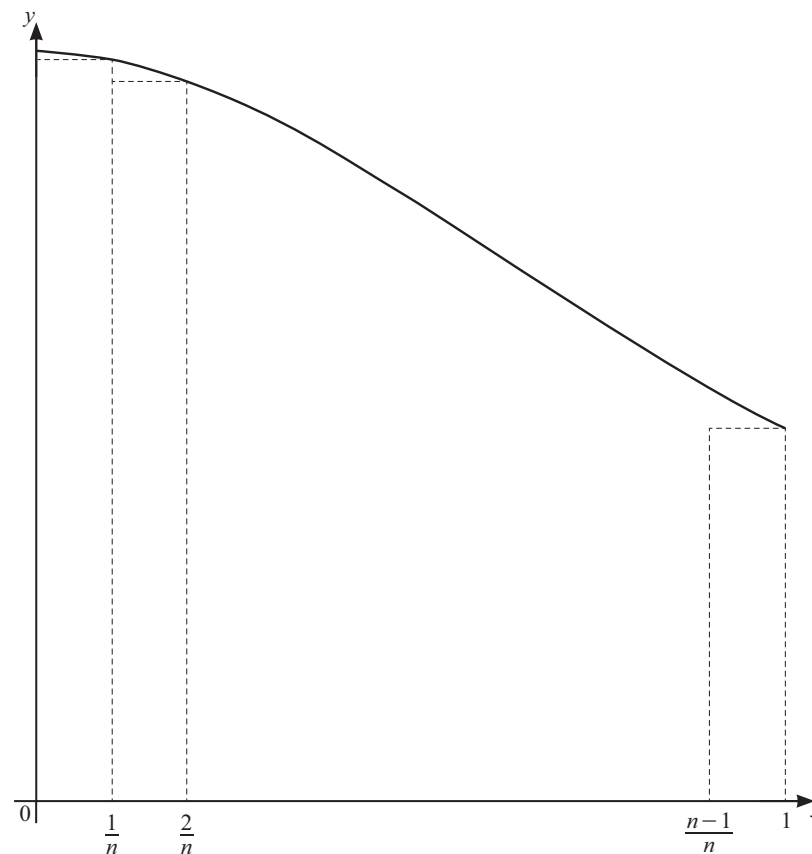
The diagram shows the curve with equation $y = \frac{1}{\sqrt{x}}e^{\sqrt{x}}$ for $x \geq 1$, together with a set of $n-1$ rectangles of unit width.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}} < \left(2 + \frac{1}{\sqrt{n}}\right) e^{\sqrt{n}} - 2e. \quad [5]$$

- (b) Use a similar method to find, in terms of n , a lower bound for $\sum_{r=1}^n \frac{1}{\sqrt{r}} e^{\sqrt{r}}$. [4]

6



The diagram shows the curve with equation $y = \frac{1}{x^2 + 1}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{n}{n^2 + r^2} < \frac{1}{4} \pi. \quad [5]$$

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(b) Use a similar method to find a lower bound for $\sum_{r=1}^n \frac{n}{n^2 + r^2}$. Give your answer in terms of n and π . [4]

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(c) Deduce the exact value of $\lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{n}{n^2 + r^2}$. [1]

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