

2(a)	$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{1 + \sinh t}{\cosh t}$
2(b)	$\frac{d}{dt} \left(\frac{1 + \sinh t}{\cosh t} \right) = \frac{\cosh t (\cosh t) - (1 + \sinh t) \sinh t}{\cosh^2 t} = \frac{1 - \sinh t}{\cosh^2 t}$
	$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{dy}{dx} \right) \times \frac{dt}{dx} = \frac{1 - \sinh t}{\cosh^3 t}$
2(c)	$y = y(0) + xy'(0) + \frac{x^2}{2!} y''(0) = 1 + x + \frac{1}{2} x^2$
	$2^n = (2n+1)I_n - 2nI_{n-1} \Rightarrow (2n+1)I_n = 2^n + 2nI_{n-1}$

3(a)	$\frac{d}{dx} (x^2 y) = x^2 y' + 2xy$
	$\frac{d}{dx} ((x+y)^3) = 3(x+y)^2 (1+y')$
	$y' - 4 + 3(1+y') = 0 \Rightarrow y' = \frac{1}{4}$
3(b)	$x^2 y'' + 2xy' + 2xy' + 2y$
	$+3(x+y)^2 y'' + 6(1+y')^2 (x+y) [= 0]$
	$(-1)^2 y'' + 4(-1)(\frac{1}{4}) + 2(2) + 3y'' + 6(\frac{5}{4})^2 [= 0]$
	$y'' = -\frac{99}{32}$

7(a)	$\frac{d}{dx} \left(\frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) \right) = -\frac{1}{2} x^2 (4-x^2)^{-\frac{1}{2}} + \frac{1}{2} (4-x^2)^{\frac{1}{2}} + 2 (4-x^2)^{-\frac{1}{2}}$
	$= \frac{1}{2} (4-x^2) (4-x^2)^{-\frac{1}{2}} + \frac{1}{2} (4-x^2)^{\frac{1}{2}} = (4-x^2)^{\frac{1}{2}}$
7(b)	$e^{\frac{1}{2} \int (2+x)^{-1} dx} = \sqrt{2+x}$
	$\frac{d}{dx} (y \sqrt{2+x}) = \sqrt{2+x} \sqrt{2-x} = \sqrt{4-x^2}$
	$y \sqrt{2+x} = \frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) [+ C]$
	$\frac{1}{2} \sqrt{3} = \frac{1}{2} \sqrt{3} + \frac{1}{3} \pi + C$
	$y \sqrt{2+x} = \frac{1}{2} x \sqrt{4-x^2} + 2 \sin^{-1} \left(\frac{1}{2} x \right) - \frac{1}{3} \pi$

7(a)(i)	$\dot{x} = 4 \frac{\sec^2 \frac{1}{2} t}{\tan \frac{1}{2} t} + 3 \operatorname{cosec}^2 t - 3$
	$\dot{x} = 8 \operatorname{cosec} t + 3 \operatorname{cosec}^2 t - 3 = 8 \operatorname{cosec} t + 3 \cot^2 t$
7(a)(ii)	$\dot{y} = 6 \operatorname{cosec} t - 4 \cot^2 t$
7(a)(iii)	$\begin{aligned} \dot{x}^2 + \dot{y}^2 &= (8 \operatorname{cosec} t + 3 \cot^2 t)^2 + (6 \operatorname{cosec} t - 4 \cot^2 t)^2 \\ &= 100 \operatorname{cosec}^2 t + 25 \cot^4 t = 100 \operatorname{cosec}^2 t + 25 (\operatorname{cosec}^2 t - 1)^2 \\ &= 25 \operatorname{cosec}^4 t + 50 \operatorname{cosec}^2 t + 25 \\ &= 25 (\operatorname{cosec}^2 t + 1)^2 \end{aligned}$
	$5 \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}^2 t + 1 dt = 5 [-\cot t + t]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = 5 \left(-\frac{1}{\sqrt{3}} + \frac{1}{3} \pi + \sqrt{3} - \frac{1}{6} \pi \right) = \frac{5}{6} \pi + \frac{10}{3} \sqrt{3}$

7(b)

$$\frac{\dot{y}}{\dot{x}} = \frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} = \frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} = \frac{6 \sec t - 4 \cot t}{8 \sec t + 3 \cot t}$$

$$\begin{aligned} & \frac{d}{dt} \left(\frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} \right) \\ &= \frac{(8 \csc t + 3 \cot^2 t)(-6 \cot t \csc t + 8 \cot t \csc^2 t) + (6 \csc t - 4 \cot^2 t)(6 \csc^2 t \cot t + 8 \cot t \csc t)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2} \end{aligned}$$

or

$$\begin{aligned} & \frac{d}{dt} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right) \\ &= \frac{(8 \sin t + 3 \cos^2 t)(6 \cos t + 8 \cos t \sin t) - (6 \sin t - 4 \cos^2 t)(8 \cos t - 6 \cos t \sin t)}{(8 \sin t + 3 \cos^2 t)^2} \end{aligned}$$

$$\frac{100 \cos t \sin^2 t + 50 \cos^3 t}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2}$$

$$\begin{aligned} \frac{d^2 y}{dx^2} &= \frac{d}{dt} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right) \times \frac{dt}{dx} = \frac{50 \sin^2 t \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^3} \\ &= \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^3} \end{aligned}$$

1

$$f'(x) = -(x+2)^{-2} e^{(x+2)^{-1}}$$

$$f''(x) = (x+2)^{-4} e^{(x+2)^{-1}} + 2(x+2)^{-3} e^{(x+2)^{-1}} = \frac{2x+5}{(x+2)^4} e^{(x+2)^{-1}}$$

$$f(0) = e^{\frac{1}{2}} \quad f'(0) = -\frac{1}{4} e^{\frac{1}{2}} \quad f''(0) = \frac{5}{16} e^{\frac{1}{2}}$$

$$e^{(x+2)^{-1}} = e^{\frac{1}{2}} - \frac{1}{4} e^{\frac{1}{2}} x + \frac{5}{32} e^{\frac{1}{2}} x^2$$