2 The curve C has parametric equations

$$x = \sinh t \quad \text{and} \quad y = t + \cosh t.$$

(a) Find $\frac{dy}{dx}$ in terms of t. [2]

(b) Show that $\frac{d^2y}{dx^2} = \frac{1 - \sinh t}{\cosh^3 t}$. [4]

(c) Find the Maclaurin's series for y in terms of x up to and including the term in x^2. [2]

3 The variables x and y are such that $y = 2$ when $x = -1$ and

$$x^2y + (x+y)^3 = 3.$$

(a) Show that $\frac{dy}{dx} = \frac{1}{4}$ when $x = -1$. [3]

(b) Find the value of $\frac{d^2y}{dx^2}$ when $x = -1$. [6]

7 **(a)** Show that $\frac{d}{dx}\left(\frac{1}{2}x\sqrt{4-x^2}+2\sin^{-1}\left(\frac{1}{2}x\right)\right)=\sqrt{4-x^2}$. [3]

(b) Find the solution of the differential equation

$$2\frac{dy}{dx}+\frac{y}{2+x}=2\sqrt{2-x}$$

for which $y=\frac{1}{2}$ when $x=1$. Give your answer in an exact form. [8]

7 The curve C has parametric equations

$$x = 8\ln(\tan\tfrac{1}{2}t) - 3\cot t - 3t, \quad y = 6\ln(\tan\tfrac{1}{2}t) + 4\cot t + 4t, \quad \text{for } \tfrac{1}{6}\pi \leqslant t \leqslant \tfrac{1}{3}\pi.$$

(a) **(i)** Show that $\dfrac{dx}{dt} = 8\operatorname{cosec} t + 3\cot^2 t$. [1]

(ii) Find $\dfrac{dy}{dt}$. [1]

(iii) Find the exact value of the length of C. [6]

(b) Show that

$$\frac{d^2y}{dx^2} = \frac{a\cot t\operatorname{cosec} t(\operatorname{cosec}^2 t + 1)}{(b\operatorname{cosec} t + c\cot^2 t)^n},$$

where a, b, c and n are integers to be determined. [5]

1 Find the Maclaurin's series for $\mathrm{e}^{\left(\frac{}{x+2}\right)}$ up to and including the term in x^2. [5]