

1 Find the values of k for which the system of equations

$$x - y + z = 0,$$

$$x + ky + 3z = 0,$$

$$x + 2y + kz = 0,$$

does not have a unique solution.

[3]

6 The matrix $\mathbf{P}$ has non-zero eigenvalues and is given by

$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

(a) State, in terms of a , the eigenvalues of $\mathbf{P}$. [1]

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(b) (i) Find $\mathbf{P}^2$ in terms of a . [1]

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(ii) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$ in terms of a . [3]

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$$x - y + 2kz = 1,$$

$$kx + y + 2z = 2,$$

$$2x - y + z = 3,$$

does not have a unique solution.

[3]

(b) Given that $k = -\frac{1}{2}$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [3]

(c) Given instead that $k = -1$, show that the system of equations in part **(a)** is also inconsistent. Interpret this situation geometrically. [4]

Additional page

If you use the following page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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8 (a) Find the set of values of a for which the system of equations

$$\begin{aligned} 5x + ay &= 3, \\ 20x - 5y &= 2, \\ 2x - 3y + z &= 1, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

(b) Given that $a = -\frac{5}{4}$, show that the system of equations in part (a) is inconsistent and interpret this situation geometrically. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

- (c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{PDP}^{-1}$. [7]

[illegible]

Additional page

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8 (a) It is given that λ is an eigenvalue of the non-singular square matrix $\mathbf{A}$, with corresponding eigenvector $\mathbf{e}$.

Show that $\mathbf{e}$ is an eigenvector of $\mathbf{A}^3$ with corresponding eigenvalue λ^3 . [2]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & 3 & 4 \\ 0 & 1 & 0 \\ 0 & -2 & 5 \end{pmatrix}.$$

(b) Show that the eigenvalues of $\mathbf{A}$ are -1 , 1 and 5 . [2]

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- [6]

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- [3]

[illegible]

Additional page

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8 (a) Find the values of a for which the system of equations

$$\begin{aligned}\frac{3}{2}x + 3y + 8z &= 1, \\ ax + 3y + 4z &= 2, \\ ay - z &= 3,\end{aligned}$$

does not have a unique solution. [3]

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The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

(b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

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(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$.

[7]

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Additional page

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