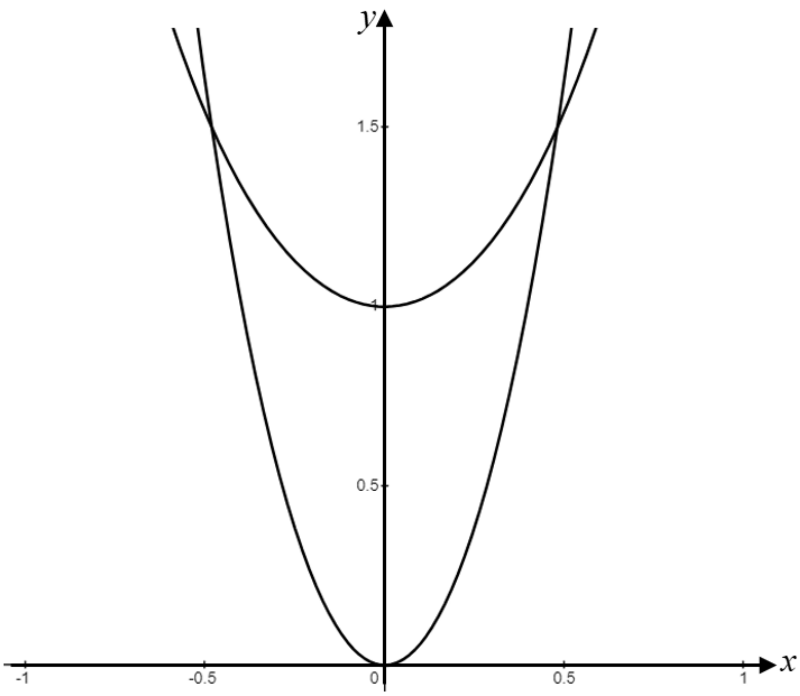
7(a)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1} x\right) = \frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$
	$= \frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{2}{(1-x)^2} = \frac{1}{1-x^2}$
	Alternative method for question 7(a)
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y\frac{\mathrm{d}y}{\mathrm{d}x} = 1$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1-\tanh^2 y} = \frac{1}{1-x^2}$
7(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{y}{x} = x\tanh^{-1} x$
	$\mathrm{e}^{-\int x^{-1}\,\mathrm{d}\theta} = \mathrm{e}^{-\ln x} = x^{-1}$
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-1}y\right) = \tanh^{-1} x$
	$x^{-1}y = x\tanh^{-1} x - \int \frac{x}{1-x^2}\,\mathrm{d}x$
	$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) + C$
	$0 = \frac{1}{2}\tanh^{-1}\frac{1}{2} + \frac{1}{2}\ln\frac{3}{4} + C \Rightarrow C = -\frac{1}{4}\ln 3 - \frac{1}{2}\ln\frac{3}{4}$
	$x^{-1}y = x\tanh^{-1} x + \frac{1}{2}\ln\left(1-x^2\right) - \frac{1}{4}\ln\frac{27}{16}$

1(b)	$I_2 = \cosh 1 - 2\sinh 1 + 2I_0$
	$I_2 = 3\cosh 1 - 2\sinh 1 - 2$
2(a)	$1 + 2\sinh^2 x = 6\sinh^2 x \Rightarrow \sinh x = \pm\frac{1}{2}$
	$x = \sinh^{-1}\left(\pm\frac{1}{2}\right) = \ln\left(\pm\frac{1}{2} + \sqrt{\frac{1}{4}+1}\right)$
	$x = \ln\left(\pm\frac{1}{2} + \frac{1}{2}\sqrt{5}\right)$

2(b)	

3(a)	$\sin 5\theta = \mathrm{Im}\left(\cos\theta + \mathrm{i}\sin\theta\right)^5 = \sin^5\theta - 10\sin^3\theta\cos^2\theta + 5\sin\theta\cos^4\theta$
	$= \sin^5\theta - 10\sin^3\theta\left(1-\sin^2\theta\right) + 5\sin\theta\left(1-\sin^2\theta\right)^2$
	$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$
3(b)	$x = \sin\theta, \quad \sin 5\theta = \frac{1}{2}\sqrt{3}$
	$5\theta = \frac{1}{3}\pi + 2k\pi$ or $5\theta = \frac{2}{3}\pi + 2k\pi$
	$\sin\left(\frac{1}{15}\pi\right)$
	$\sin(\frac{2}{15}\pi), \sin(-\frac{5}{15}\pi), \sin(-\frac{4}{15}\pi), \sin(\frac{7}{15}\pi)$

4(a)	$\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x < \frac{1}{2}\left(\left(\frac{1}{N}\right)3^{\frac{1}{N}} + \left(\frac{1}{N}\right)3^{\frac{2}{N}} + \cdots + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}} + \left(\frac{1}{N}\right)3^{\frac{N}{N}}\right)$
	$= \frac{1}{2N}\sum_{n=1}^{N}\left(3^{\frac{1}{N}}\right)^n = \frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}}-1\right)}$
4(b)	$\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x > \frac{1}{2}\left(\left(\frac{1}{N}\right) + \left(\frac{1}{N}\right)3^{\frac{1}{N}} + \cdots + \left(\frac{1}{N}\right)3^{\frac{N-2}{N}} + \left(\frac{1}{N}\right)3^{\frac{N-1}{N}}\right)$
	$= \frac{1}{2N}\sum_{n=0}^{N-1}\left(3^{\frac{1}{N}}\right)^n = \frac{1}{N\left(3^{\frac{1}{N}}-1\right)}$
4(c)	$\frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}}-1\right)} - \frac{1}{N\left(3^{\frac{1}{N}}-1\right)} = \frac{1}{N}$
	$\frac{1}{N} \to 0$ as $N \to \infty$.

5(a)	$m^2+m+1=0 \Rightarrow m=-\frac{1}{2}\pm\frac{1}{2}\sqrt{3}$
	$y=\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x\right)$
	$y=p\sin 2x+q\cos 2x \Rightarrow y'=2p\cos 2x-2q\sin 2x \Rightarrow y''=-4p\sin 2x-4q\cos 2x$
	$-4p-2q+p=1$ $\quad -4q+2p+q=1$
	$p=-\frac{1}{13}$ $\quad q=-\frac{5}{13}$
	$y=\mathrm{e}^{-\frac{1}{2}x}\left(A\cos\frac{\sqrt{3}}{2}x+B\sin\frac{\sqrt{3}}{2}x\right)-\frac{1}{13}\sin 2x-\frac{5}{13}\cos 2x$
5(b)	$y\approx-\frac{1}{13}\sin 2x-\frac{5}{13}\cos 2x=\sqrt{\frac{2}{13}}\left(-\frac{1}{\sqrt{26}}\sin 2x-\frac{5}{\sqrt{26}}\cos 2x\right)$
	$\sqrt{\frac{2}{13}}\sin(2x-1.77)$

6	$\int \frac{1+x}{1-2x-x^2}\,\mathrm{d}x = -\frac{1}{2}\ln\left(1-2x-x^2\right)$
	$\mathrm{e}^{-\frac{1}{2}\ln\left(1-2x-x^2\right)} = \frac{1}{\sqrt{1-2x-x^2}}$
	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{y}{\sqrt{1-2x-x^2}}\right) = \frac{1}{\sqrt{1-2x-x^2}}$
	$1-2x-x^2 = 2-\left(x+1\right)^2$
	$\int \frac{1}{\sqrt{2-\left(x+1\right)^2}}\,\mathrm{d}x = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+C$
	$\pi = \sin^{-1}\frac{1}{\sqrt{2}}+C \Rightarrow C = \frac{3}{4}\pi$
	$\frac{y}{\sqrt{1-2x-x^2}} = \sin^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+\frac{3}{4}\pi$

7(a)(i)	$\dot{x} = 4\dfrac{\sec^2 \frac{1}{2}t}{\tan \frac{1}{2}t} + 3\operatorname{cosec}^2 t - 3$ $\dot{x} = 8\operatorname{cosec} t + 3\operatorname{cosec}^2 t - 3 = 8\operatorname{cosec} t + 3\cot^2 t$
7(a)(ii)	$\dot{y} = 6\operatorname{cosec} t - 4\cot^2 t$
7(a)(iii)	$\dot{x}^2 + \dot{y}^2 = \left(8\operatorname{cosec} t + 3\cot^2 t\right)^2 + \left(6\operatorname{cosec} t - 4\cot^2 t\right)^2$ $= 100\operatorname{cosec}^2 t + 25\cot^4 t = 100\operatorname{cosec}^2 t + 25\left(\operatorname{cosec}^2 t - 1\right)^2$ $= 25\operatorname{cosec}^4 t + 50\operatorname{cosec}^2 t + 25$
	$= 25\left(\operatorname{cosec}^2 t + 1\right)^2$
	$5\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}^2 t + 1\,\mathrm{d}t = 5\left[-\cot t + t\right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = 5\left(-\frac{1}{\sqrt{3}} + \frac{1}{3}\pi + \sqrt{3} - \frac{1}{6}\pi\right) = \frac{5}{6}\pi + \frac{10}{3}\sqrt{3}$

7(b)	$\frac{\dot{y}}{\dot{x}}=\frac{6\operatorname{cosec}t-4\cot^2 t}{8\operatorname{cosec}t+3\cot^2 t}=\frac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}=\frac{6\sec t-4\cot t}{8\sec t+3\cot t}$
	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{6\operatorname{cosec}t-4\cot^2 t}{8\operatorname{cosec}t+3\cot^2 t}\right)$ $=\frac{\left(8\csc t+3\cot^2 t\right)\left(-6\cot t\csc t+8\cot t\csc^2 t\right)+\left(6\csc t-4\cot^2 t\right)\left(6\csc^2 t\cot t+8\cot t\csc t\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^2}$ or $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}\right)$ $=\frac{\left(8\sin t+3\cos^2 t\right)\left(6\cos t+8\cos t\sin t\right)-\left(6\sin t-4\cos^2 t\right)\left(8\cos t-6\cos t\sin t\right)}{\left(8\sin t+3\cos^2 t\right)^2}$
	$\frac{100\cos t\sin^2 t+50\cos^3 t}{\left(8\sin t+3\cos^2 t\right)^2}=\frac{50\cos t\left(\sin^2 t+1\right)}{\left(8\sin t+3\cos^2 t\right)^2}=\frac{50\cot t\operatorname{cosec}t\left(\operatorname{cosec}^2 t+1\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^2}$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{6\sin t-4\cos^2 t}{8\sin t+3\cos^2 t}\right)\times\frac{\mathrm{d}t}{\mathrm{d}x}=\frac{50\sin^2 t\cos t\left(\sin^2 t+1\right)}{\left(8\sin t+3\cos^2 t\right)^3}$ $=\frac{50\cot t\operatorname{cosec}t\left(\operatorname{cosec}^2 t+1\right)}{\left(8\operatorname{cosec}t+3\cot^2 t\right)^3}$

8(a)	$\begin{vmatrix}5 & a & 0\\20 & -5 & 0\\2 & -3 & 1\end{vmatrix}=-20a-25$
	$a\neq-\frac{5}{4}$
	Three planes intersect at a *single* point.

6(a)	$\tanh u = \dfrac{e^{u} - e^{-u}}{e^{u} + e^{-u}} \quad \operatorname{sech} u = \dfrac{2}{e^{u} + e^{-u}}$
	$1 - \left(\dfrac{e^{u} - e^{-u}}{e^{u} + e^{-u}}\right)^2 = \dfrac{\left(e^{u} + e^{-u}\right)^2 - \left(e^{u} - e^{-u}\right)^2}{\left(e^{u} + e^{-u}\right)^2} = \dfrac{4}{\left(e^{u} + e^{-u}\right)^2}$
	$= \operatorname{sech}^2 u$
6(b)	$u = \operatorname{sech}^{-1} t \Rightarrow \operatorname{sech} u = t \Rightarrow -\tanh u \operatorname{sech} u \dfrac{du}{dt} = 1$
	$\dfrac{du}{dt} = -\dfrac{1}{\tanh u \operatorname{sech} u} = -\dfrac{1}{t\sqrt{1-t^2}}$
	First alternative method for question 6(b)
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \dfrac{1}{t} \Rightarrow \sinh u \dfrac{du}{dt} = -\dfrac{1}{t^2}$
	$\dfrac{du}{dt} = -\dfrac{1}{t^2\sqrt{\cosh^2 u - 1}} = -\dfrac{1}{t^2\sqrt{t^{-2} - 1}} = -\dfrac{1}{t\sqrt{1-t^2}}$

6(b)	**Second alternative method for question 6(b)**
	$u = \operatorname{sech}^{-1} t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \cosh^{-1}\left(\frac{1}{t}\right)$
	$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{1}{\sqrt{\left(\frac{1}{t}\right)^2 - 1}} \times \left(-t^{-2}\right) = -\frac{1}{t\sqrt{1-t^2}}$
	Third alternative method for question 6(b)
	$\operatorname{sech} u = t \Rightarrow \cosh u = \frac{1}{t} \Rightarrow u = \ln\left(\frac{1}{t} + \sqrt{\frac{1}{t^2} - 1}\right) = \ln\left(\frac{1+\sqrt{1-t^2}}{t}\right)$
	$\frac{\mathrm{d}u}{\mathrm{d}t} = \left(\frac{-1-\sqrt{1-t^2}}{t^2\sqrt{1-t^2}}\right)\left(\frac{t}{1+\sqrt{1-t^2}}\right) = -\frac{1}{t\sqrt{1-t^2}}$
6(c)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \operatorname{sech}^{-1} t - \frac{1}{\sqrt{1-t^2}}$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{1-t^2}$
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \times \frac{\mathrm{d}t}{\mathrm{d}x} = -\sqrt{1-t^2} + \left(1-t^2\right)\operatorname{sech}^{-1} t$

6(d)	$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{t}{\sqrt{1-t^2}} - \frac{1-t^2}{t\sqrt{1-t^2}} - 2t\operatorname{sech}^{-1} t$
	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = t\sqrt{1-t^2} - \frac{\left(1-t^2\right)^{\frac{3}{2}}}{t} - 2t\left(1-t^2\right)\operatorname{sech}^{-1} t$

2(a)	$\tanh t = \dfrac{e^t - e^{-t}}{e^t + e^{-t}} \quad \operatorname{sech} t = \dfrac{2}{e^t + e^{-t}}$
	$\left(\dfrac{e^t - e^{-t}}{e^t + e^{-t}}\right)^2 + \dfrac{4}{\left(e^t + e^{-t}\right)^2} = \dfrac{e^{2t} + e^{-2t} + 2}{\left(e^t + e^{-t}\right)^2} = 1$
2(b)	$\dfrac{dx}{dt} = \tanh t$
	$\dfrac{dy}{dt} = \dfrac{\cosh t}{1 + \sinh^2 t} = \operatorname{sech} t$
	$\int_0^1 \sqrt{\tanh^2 t + \operatorname{sech}^2 t}\ dt = \int_0^1 1\, dt$
	$= 1$

3(a)	$\dfrac{d}{dx}\left(9y^2\right) = 18yy'$
	$\dfrac{d}{dx}\left(-3\sinh^{-1}(xy)\right) = -3\dfrac{xy' + y}{\sqrt{1 + x^2y^2}}$
	$\frac{18}{3}y' - 3\dfrac{4y' + \frac{1}{3}}{\frac{5}{3}} = 0 \Rightarrow y' = -\frac{1}{2}$

3(b)	$18yy''+18(y')^2$
	$-3\left(\frac{\sqrt{1+x^2y^2}\,(xy''+2y')-(xy'+y)^2(1+x^2y^2)^{-\frac{1}{2}}(xy)}{1+x^2y^2}\right)$
	$\frac{18}{3}y''+\frac{18}{4}-3\frac{\frac{5}{3}(4y''-1)-\left(\frac{5}{3}\right)^2\left(\frac{3}{5}\right)\left(\frac{4}{3}\right)}{\frac{25}{9}}=0$ $\frac{18}{3}y''+\frac{18}{4}-\frac{45(4y''-1)-60}{25}=0$
	$-\frac{6}{5}y''+\frac{87}{10}=0\Rightarrow y''=\frac{29}{4}$