

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

[9]

2 (a) Find the exact values of x for which $\cosh 2x = 6 \sinh^2 x$, giving your answers in logarithmic form. [4]

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(b) Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6 \sinh^2 x$ on the same diagram. [2]

6 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u.$$
 [3]

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(b) Show that $\frac{d}{dt}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}.$ [4]

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It is given that

$$x = \tanh^{-1}t \quad \text{and} \quad y = t \operatorname{sech}^{-1}t, \quad \text{for } 0 < t < 1.$$

- (c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1}t$. [4]

[illegible]

- (d) Find $\frac{d^2y}{dx^2}$ in terms of t . [4]

5

- 2 (a) Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$\tanh^2 t + \operatorname{sech}^2 t = 1. \quad [3]$$

[illegible]

- (b) The curve C has parametric equations

$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leq t \leq 1.$$

Find the length of C . [5]

This image shows a full page of handwriting practice paper. It features multiple rows of horizontal lines. Each row consists of a solid top line, a dashed middle line, and a solid bottom line, providing a guide for letter height and placement. The paper is otherwise blank, with no text or markings other than the printed lines.

3 The curve C has equation

$$9y^2 - 3 \sinh^{-1}(xy) = 1 - 3 \ln 3.$$

(a) Show that, at the point $(4, \frac{1}{3})$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [4]

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(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(4, \frac{1}{3})$. [5]

⑦

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