7 **(a)** Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$. [3]

(b) Find the solution of the differential equation

$$x\frac{dy}{dx} - y = x^2 \tanh^{-1}x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

2 **(a)** Find the exact values of x for which $\cosh 2x = 6\sinh^2 x$, giving your answers in logarithmic form. [4]

(b) Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6\sinh^2 x$ on the same diagram. [2]

6 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$1 - \tanh^2 u = \operatorname{sech}^2 u.$$ [3]

(b) Show that $\frac{\mathrm{d}}{\mathrm{d}t}(\operatorname{sech}^{-1} t) = -\frac{1}{t\sqrt{1-t^2}}$. [4]

It is given that

$$x = \tanh^{-1} t \quad \text{and} \quad y = t\,\text{sech}^{-1} t, \quad \text{for } 0 < t < 1.$$

(c) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -\sqrt{1-t^2} + \left(1-t^2\right)\text{sech}^{-1} t$. [4]

(d) Find $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$ in terms of t. [4]

2 **(a)** Starting from the definitions of tanh and sech in terms of exponentials, prove that

$$\tanh^2 t + \text{sech}^2 t = 1.$$ [3]

(b) The curve C has parametric equations

$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leqslant t \leqslant 1.$$

Find the length of C. [5]

3 The curve C has equation

$$9y^2 - 3\sinh^{-1}(xy) = 1 - 3\ln 3.$$

(a) Show that, at the point $(4, \frac{1}{3})$ on C, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{2}$. [4]

(b) Find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at the point $(4, \frac{1}{3})$. [5]