

7 (a) Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$.

[3]

(b) Find the solution of the differential equation

$$x \frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

- 2 (a)** Find the exact values of x for which $\cosh 2x = 6 \sinh^2 x$, giving your answers in logarithmic form.

This image shows a full page of white paper with ten horizontal rows of small black dots, used as guides for handwriting practice. The dots are arranged in straight, parallel lines across the entire width of the page.

- (b)** Sketch the curves $C_1 : y = \cosh 2x$ and $C_2 : y = 6 \sinh^2 x$ on the same diagram. [2]

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[3]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

[4]

[illegible]

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It is given that

$$x = \tanh^{-1}t \quad \text{and} \quad y = t \operatorname{sech}^{-1}t, \quad \text{for } 0 < t < 1.$$

(c) Show that $\frac{dy}{dx} = -\sqrt{1-t^2} + (1-t^2)\operatorname{sech}^{-1}t$. [4]

[illegible]

(d) Find $\frac{d^2y}{dx^2}$ in terms of t . [4]

Handwriting practice lines (dotted lines on a solid background) for the letter 'e'.

Name:

- 2

[3]

[illegible]

- (b)

$$x = \ln(\cosh t), \quad y = \tan^{-1}(\sinh t), \quad \text{for } 0 \leq t \leq 1.$$

[5]

Handwriting practice lines (dotted lines on a solid background) for the letter 'a'.

3 The curve C has equation

$$9y^2 - 3 \sinh^{-1}(xy) = 1 - 3 \ln 3.$$

(a) Show that, at the point $(4, \frac{1}{3})$ on C , $\frac{dy}{dx} = -\frac{1}{2}$. [4]

(b)

$$\frac{d^2y}{dx^2}$$

at the point $(4, \frac{1}{3})$.

[5]