

3	$\frac{d}{dx}(x \cos x) = \cos x - x \sin x = (-1)^1(x \sin x - (2(1) - 1) \cos x)$
	Assume true for $n = k$, so $\frac{d^{2k-1}}{dx^{2k-1}}(x \cos x) = (-1)^k(x \sin x - (2k - 1) \cos x)$
	Then $\frac{d^{2k}}{dx^{2k}}(x \cos x) = (-1)^k(x \cos x + 2k \sin x)$
	$\frac{d^{2k+1}}{dx^{2k+1}}(x \cos x) = (-1)^k(-x \sin x + \cos x + 2k \cos x)$ $= (-1)^{k+1}(x \sin x - (2k + 1) \cos x)$
	So, it is also true for $n = k + 1$. Hence, [by induction], $\frac{d^{2n-1}}{dx^{2n-1}}(x \cos x) = (-1)^n(x \sin x - (2n - 1) \cos x)$ is true for all positive integers.

3(a)	$u_1 < 5$ (given)
	Assume that $u_k < 5$ for some positive integer k .
	Then $5 - u_{k+1} = 5 - \frac{6u_k + 5}{u_k + 2} = \frac{5u_k + 10 - 6u_k - 5}{u_k + 2}$
	$= \frac{5 - u_k}{u_k + 2} > 0.$
	Hence, by induction, $u_n < 5$ for all positive integers n .
3(b)	$u_{n+1} - u_n = \frac{6u_n + 5}{u_n + 2} - u_n = \frac{6u_n + 5 - u_n^2 - 2u_n}{u_n + 2} = \frac{-u_n^2 + 4u_n + 5}{u_n + 2}$
	$= \frac{(5 - u_n)(1 + u_n)}{u_n + 2}.$ So $u_n < 5 \Rightarrow u_{n+1} - u_n > 0$

1	$\text{LHS} = \left(\frac{6}{5}\right)^1 = \frac{6}{5} \quad \text{RHS} = 1 + \frac{1}{5} = \frac{6}{5}$
	Assume that $\left(\frac{6}{5}\right)^k \geq 1 + \frac{1}{5}k$ for some positive integer k .
	Then $\left(\frac{6}{5}\right)^{k+1} \geq \left(\frac{6}{5}\right)\left(1 + \frac{1}{5}k\right)$
	$= \frac{6}{5} + \frac{6}{25}k \geq \frac{6}{5} + \frac{1}{5}k = 1 + \frac{1}{5}(k+1)$
	Hence, by induction, $\left(\frac{6}{5}\right)^n \geq 1 + \frac{1}{5}n$ is true for every positive integer n .

3(a)	$u_1 = 5 = 6^1 - 1$
	Assume that it is true for $n = k$, so $u_k = 6^k - 1$.
	Then $u_{k+1} = 6(6^k - 1) + 5 = 6^{k+1} - 1$.
	So, it is also true for $n = k + 1$. Hence, by induction, $u_n = 6^n - 1$ is true for all positive integers.
3(b)	$\frac{u_{2n}}{u_n} = \frac{6^{2n} - 1}{6^n - 1} = \frac{(6^n - 1)(6^n + 1)}{6^n - 1} = 6^n + 1$

2	$2025 + 47 - 2 = 2070 = 45 \times 46$ is divisible by 46.
	Assume that $2025^k + 47^k - 2$ is divisible by 46 for some positive integer k .
	Then $2025^{k+1} + 47^{k+1} - 2 = (2024 + 1)2025^k + (46 + 1)47^k - 2$
	is divisible by 46 because $2024 \times 2025^k + 46 \times 47^k = 46(44 \times 2025^k) + 46 \times 47^k$ is divisible by 46
	Hence, [by induction], true for every positive integer n .