

3 Prove by mathematical induction that, for every positive integer n ,

$$\frac{\mathrm{d}^{2n-1}}{\mathrm{d}x^{2n-1}}(x\cos x) = (-1)^n(x\sin x - (2n-1)\cos x).$$

[7]

3 The sequence of positive numbers $u_1, u_2, u_3, \dots$ is such that $u_1 < 5$ and, for $n \geq 1$,

$$u_{n+1} = \frac{6u_n + 5}{u_n + 2}.$$

(a) By considering $5 - u_{n+1}$, prove by mathematical induction that $u_n < 5$ for all positive integers n . [5]

(b) Show that $u_{n+1} > u_n$ for $n \geq 1$. [3]

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Prove by mathematical induction that $\left(\frac{n}{5}\right) \geq 1 + \frac{1}{5}n$ for all positive integers n .

[5]

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- 3** The sequence $u_1, u_2, u_3, \dots$ is such that $u_1 = 5$ and $u_{n+1} = 6u_n + 5$ for $n \geq 1$.

- (a) Prove by induction that $u_n = 6^n - 1$ for all positive integers n .

[5]

- (b) Deduce that u_{2n} is divisible by u_n for $n \geq 1$.

[2]

- 2 Prove by mathematical induction that $2025^n + 47^n - 2$ is divisible by 46 for all positive integers n . [6]

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