5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$
	$5(-3)-2(-1)+(-1)=-14$
	$5x-2y+z=-14$
5(b)	$\dfrac{1}{\sqrt{5^2+2^2+1^2}}\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$
	$\frac{7}{\sqrt{30}}$
5(c)	States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$.
	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$
	$r = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

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6(a)	$\dfrac{1}{\sqrt{1^2+3^2+2^2}} = \dfrac{1}{\sqrt{14}}$

6(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$
	$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \sqrt{21}\sqrt{14}\cos\alpha \Rightarrow \cos\alpha = \dfrac{17}{\sqrt{21}\sqrt{14}}$
	$7.5°$

6(c)	$\overrightarrow{OP}=\lambda\begin{pmatrix}2\\-1\\-1\end{pmatrix},\overrightarrow{OQ}=\begin{pmatrix}2\mu\\-1+\mu\\2-3\mu\end{pmatrix}\Rightarrow\overrightarrow{PQ}=\begin{pmatrix}2\mu-2\lambda\\-1+\mu+\lambda\\2-3\mu+\lambda\end{pmatrix}$
	$\begin{pmatrix}2\mu-2\lambda\\-1+\mu+\lambda\\2-3\mu+\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\-1\\-1\end{pmatrix}=0$
	$6\mu-6\lambda=1$
	$\begin{pmatrix}2\mu-2\lambda\\-1+\mu+\lambda\\2-3\mu+\lambda\end{pmatrix}\bullet\begin{pmatrix}2\\1\\-3\end{pmatrix}=0\Rightarrow 14\mu-6\lambda=7$
	$\mu=\dfrac{3}{4}\Rightarrow\overrightarrow{OQ}=\begin{pmatrix}\frac{3}{2}\\-\frac{1}{4}\\-\frac{1}{4}\end{pmatrix}=\frac{1}{4}\begin{pmatrix}6\\-1\\-1\end{pmatrix}.$
	$\mathbf{r}=\frac{1}{4}\begin{pmatrix}6\\-1\\-1\end{pmatrix}+k\begin{pmatrix}1\\1\\1\end{pmatrix}$

4(a)	$\overrightarrow{AB}=-\mathbf{i}-3\mathbf{j}-3\mathbf{k}\quad\overrightarrow{AC}=-\mathbf{i}-4\mathbf{j}-6\mathbf{k}$
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-1&-3&-3\\-1&-4&-6\end{vmatrix}=\begin{pmatrix}6\\-3\\1\end{pmatrix}$
	$6(3)-3(5)+(5)=8\Rightarrow 6x-3y+z=8$
4(b)	$\overrightarrow{CD}=\begin{pmatrix}1\\-1\\3\end{pmatrix}-\begin{pmatrix}2\\1\\-1\end{pmatrix}=\begin{pmatrix}-1\\-2\\4\end{pmatrix}$
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-1&-3&-3\\-1&-2&4\end{vmatrix}=\begin{pmatrix}-18\\7\\-1\end{pmatrix}$
	$\dfrac{1}{\sqrt{374}}\left\lvert\begin{pmatrix}-1\\-4\\-6\end{pmatrix}\bullet\begin{pmatrix}-18\\7\\-1\end{pmatrix}\right\rvert=\dfrac{4}{\sqrt{374}}=0.207$

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6(a)	$\overrightarrow{AB}=2\mathbf{j}+4\mathbf{k}\quad\overrightarrow{AC}=\mathbf{i}-\mathbf{j}+\mathbf{k}$
	$\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\0&2&4\\1&-1&1\end{vmatrix}=\begin{pmatrix}6\\4\\-2\end{pmatrix}\sim\begin{pmatrix}3\\2\\-1\end{pmatrix}$
	$3(1)+2(0)-(-2)=5\Rightarrow 3x+2y-z=5$

6(b)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 0 & 0 & t+2 \end{vmatrix} = \begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$
	$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{14}\cos\theta$
	$(t \neq -2) \Rightarrow \theta = 36.7^\circ$
6(c)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ -1 & 1 & t+1 \end{vmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix}$
	$\left\lvert \frac{1}{\sqrt{t^2-2t+6}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix} \right\rvert = \left\lvert \frac{t+2}{\sqrt{t^2-2t+6}} \right\rvert$
	$\left\lvert \frac{t+2}{\sqrt{t^2-2t+6}} \right\rvert = \sqrt{2} \Rightarrow (t+2)^2 = 2\left(t^2-2t+6\right)$
	$t^2 - 8t + 8 = 0 \Rightarrow t = 2\left(2+\sqrt{2}\right), t = 2\left(2-\sqrt{2}\right)$

5(a)	$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}$
	$6(2) - (3) + 8(-2) = -7$
	$6x - y + 8z = -7$

5(b)	$\overrightarrow{OF} = \overrightarrow{OP} + \overrightarrow{PF} = \begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix} + t\begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 4+6t \\ 2-t \\ 9+8t \end{pmatrix}$
	$6(4+6t) - (2-t) + 8(9+8t) = -7 \Rightarrow 101t + 94 = -7$
	$t = -1 \Rightarrow \overrightarrow{OF} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$
5(c)	$\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \sqrt{35}\sqrt{101}\cos\alpha \Rightarrow \cos\alpha = \frac{5}{\sqrt{35}\sqrt{101}}$
	Acute angle between l and Π is $90 - \alpha = 4.8^\circ$