

|      |                                                                                                                                                                                              |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a) | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 2 \\ 1 & 3 & 1 \end{vmatrix} = \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix} \sim \begin{pmatrix} 5 \\ -2 \\ 1 \end{pmatrix}$ |
|      | $5(-3) - 2(-1) + (-1) = -14$                                                                                                                                                                 |
|      | $5x - 2y + z = -14$                                                                                                                                                                          |
|      |                                                                                                                                                                                              |
| 5(b) | $\frac{1}{\sqrt{5^2 + 2^2 + 1^2}} \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \\ -1 \end{pmatrix}$                                                             |
|      | $\frac{7}{\sqrt{30}}$                                                                                                                                                                        |
|      |                                                                                                                                                                                              |
| 5(c) | States point common to both planes e.g. $\begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix}$ .                                                                                                        |
|      | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & -2 & 1 \\ 3 & 2 & -1 \end{vmatrix} = \begin{pmatrix} 0 \\ 8 \\ 16 \end{pmatrix} \sim \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ |
|      | $r = \begin{pmatrix} 0 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$                                                                                          |
|      |                                                                                                                                                                                              |

|      |                                                          |
|------|----------------------------------------------------------|
| 6(a) | $\frac{1}{\sqrt{1^2 + 3^2 + 2^2}} = \frac{1}{\sqrt{14}}$ |
|      |                                                          |

6(b)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 2 \\ 2 & 0 & -1 \end{vmatrix} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = \sqrt{21}\sqrt{14} \cos \alpha \Rightarrow \cos \alpha = \frac{17}{\sqrt{21}\sqrt{14}}$$

7.5°

6(c)

$$\overrightarrow{OP} = \lambda \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}, \overrightarrow{OQ} = \begin{pmatrix} 2\mu \\ -1 + \mu \\ 2 - 3\mu \end{pmatrix} \Rightarrow \overrightarrow{PQ} = \begin{pmatrix} 2\mu - 2\lambda \\ -1 + \mu + \lambda \\ 2 - 3\mu + \lambda \end{pmatrix}$$

$$\begin{pmatrix} 2\mu - 2\lambda \\ -1 + \mu + \lambda \\ 2 - 3\mu + \lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 0$$

$$6\mu - 6\lambda = 1$$

$$\begin{pmatrix} 2\mu - 2\lambda \\ -1 + \mu + \lambda \\ 2 - 3\mu + \lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = 0 \Rightarrow 14\mu - 6\lambda = 7$$

$$\mu = \frac{3}{4} \Rightarrow \overrightarrow{OQ} = \begin{pmatrix} \frac{3}{2} \\ -\frac{1}{4} \\ -\frac{1}{4} \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix}.$$

$$\mathbf{r} = \frac{1}{4} \begin{pmatrix} 6 \\ -1 \\ -1 \end{pmatrix} + k \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

|      |                                                                                                                                                                      |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 4(a) | $\overrightarrow{AB} = -\mathbf{i} - 3\mathbf{j} - 3\mathbf{k} \quad \overrightarrow{AC} = -\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$                                  |
|      | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -4 & -6 \end{vmatrix} = \begin{pmatrix} 6 \\ -3 \\ 1 \end{pmatrix}$                    |
|      | $6(3) - 3(5) + (5) = 8 \Rightarrow 6x - 3y + z = 8$                                                                                                                  |
|      |                                                                                                                                                                      |
| 4(b) | $\overrightarrow{CD} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ 4 \end{pmatrix}$        |
|      | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & -3 \\ -1 & -2 & 4 \end{vmatrix} = \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix}$                   |
|      | $\frac{1}{\sqrt{374}} \left  \begin{pmatrix} -1 \\ -4 \\ -6 \end{pmatrix} \cdot \begin{pmatrix} -18 \\ 7 \\ -1 \end{pmatrix} \right  = \frac{4}{\sqrt{374}} = 0.207$ |
|      |                                                                                                                                                                      |

|      |                                                                                                                                                                                              |
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| 6(a) | $\overrightarrow{AB} = 2\mathbf{j} + 4\mathbf{k} \quad \overrightarrow{AC} = \mathbf{i} - \mathbf{j} + \mathbf{k}$                                                                           |
|      | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 1 & -1 & 1 \end{vmatrix} = \begin{pmatrix} 6 \\ 4 \\ -2 \end{pmatrix} \sim \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ |
|      | $3(1) + 2(0) - (-2) = 5 \Rightarrow 3x + 2y - z = 5$                                                                                                                                         |
|      |                                                                                                                                                                                              |

|      |                                                                                                                                                                                                     |
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| 6(b) | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ 0 & 0 & t+2 \end{vmatrix} = \begin{pmatrix} 2t+4 \\ 0 \\ 0 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$      |
|      | $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \sqrt{14} \cos \theta$                                                                                |
|      | $(t \neq -2) \Rightarrow \theta = 36.7^\circ$                                                                                                                                                       |
|      |                                                                                                                                                                                                     |
| 6(c) | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 4 \\ -1 & 1 & t+1 \end{vmatrix} = \begin{pmatrix} 2t-2 \\ -4 \\ 2 \end{pmatrix} \sim \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix}$ |
|      | $\left  \frac{1}{\sqrt{t^2-2t+6}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} t-1 \\ -2 \\ 1 \end{pmatrix} \right  = \left  \frac{t+2}{\sqrt{t^2-2t+6}} \right $               |
|      | $\left  \frac{t+2}{\sqrt{t^2-2t+6}} \right  = \sqrt{2} \Rightarrow (t+2)^2 = 2(t^2-2t+6)$                                                                                                           |
|      | $t^2 - 8t + 8 = 0 \Rightarrow t = 2(2 + \sqrt{2}), t = 2(2 - \sqrt{2})$                                                                                                                             |
|      |                                                                                                                                                                                                     |

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|------|------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a) | $\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix}$ |
|      | $6(2) - (3) + 8(-2) = -7$                                                                                                                      |
|      | $6x - y + 8z = -7$                                                                                                                             |
|      |                                                                                                                                                |

|      |                                                                                                                                                                                                                  |
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| 5(b) | $\overrightarrow{OF} = \overrightarrow{OP} + \overrightarrow{PF} = \begin{pmatrix} 4 \\ 2 \\ 9 \end{pmatrix} + t \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \begin{pmatrix} 4+6t \\ 2-t \\ 9+8t \end{pmatrix}$ |
|      | $6(4+6t) - (2-t) + 8(9+8t) = -7 \Rightarrow 101t + 94 = -7$                                                                                                                                                      |
|      | $t = -1 \Rightarrow \overrightarrow{OF} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$                                                                                                                            |
|      |                                                                                                                                                                                                                  |
| 5(c) | $\begin{pmatrix} 3 \\ 5 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -1 \\ 8 \end{pmatrix} = \sqrt{35}\sqrt{101}\cos\alpha \Rightarrow \cos\alpha = \frac{5}{\sqrt{35}\sqrt{101}}$                             |
|      | <p>Acute angle between <math>l</math> and <math>l'</math> is <math>90 - \alpha = 4.8^\circ</math></p>                                                                                                            |
|      |                                                                                                                                                                                                                  |