5 The plane Π_1 has equation $\mathbf{r} = -3\mathbf{i} - \mathbf{j} - \mathbf{k} + \lambda(\mathbf{j} + 2\mathbf{k}) + \mu(\mathbf{i} + 3\mathbf{j} + \mathbf{k})$.

(a) Find an equation for Π_1 in the form $ax + by + cz = d$. [4]

(b) Find the perpendicular distance from the point with position vector $-\mathbf{i} - 2\mathbf{k}$ to Π_1. [3]

(c) The plane Π_2 has equation $3x + 2y - z = 14$.

Find a vector equation of the line of intersection of Π_1 and Π_2. [4]

6 The plane Π has equation $x+3y+2z=1$.

(a) Find the perpendicular distance from the origin O to the plane Π. [2]

Relative to O, the points A, B, C have position vectors

$$-\mathbf{j}+2\mathbf{k}, \quad 2\mathbf{i}-\mathbf{k}, \quad 2\mathbf{i}-\mathbf{j}-\mathbf{k},$$

respectively.

(b) Find the acute angle between the planes OAB and Π. [4]

(c) Find an equation for the common perpendicular to the lines OC and AB. [8]

4 The points A, B, C have position vectors

$$3\mathbf{i}+5\mathbf{j}+5\mathbf{k}, \qquad 2\mathbf{i}+2\mathbf{j}+2\mathbf{k}, \qquad 2\mathbf{i}+\mathbf{j}-\mathbf{k},$$

respectively, relative to the origin O.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

(b) The point D has position vector $\mathbf{i}-\mathbf{j}+3\mathbf{k}$.

Find the shortest distance between the lines AB and CD. [5]

6 The points A, B, C have position vectors

$$\mathbf{i}-2\mathbf{k}, \quad \mathbf{i}+2\mathbf{j}+2\mathbf{k}, \quad 2\mathbf{i}-\mathbf{j}-\mathbf{k},$$

respectively.

(a) Find the equation of the plane ABC, giving your answer in the form $ax+by+cz=d$. [5]

A point D has position vector $\mathbf{i}+t\mathbf{k}$, where $t \neq -2$.

(b) Find the acute angle between the planes ABC and ABD. [4]

(c) Find the values of t such that the shortest distance between the lines AB and CD is $\sqrt{2}$. [7]

5 The plane Π has equation $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - \mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$.

(a) Find a Cartesian equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

The point P has position vector $4\mathbf{i} + 2\mathbf{j} + 9\mathbf{k}$.

(b) Find the position vector of the foot of the perpendicular from P to Π. [4]

(c) The line l is parallel to the vector $3\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

Find the acute angle between l and Π. [3]