

6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leq \theta \leq \frac{1}{3}\pi$.
(a) Sketch C and state the equation of the line of symmetry. [3]

(b) Find the exact value of the area of the region enclosed by C . [4]

In parts (c) and (d) you may use the identity $\sin 3\theta \equiv 3 \sin \theta - 4 \sin^3 \theta$.

(c) Find the maximum distance of a point on C from the initial line. [5]

(d) Find a Cartesian equation for C . [3]

5 The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leq \theta \leq \frac{1}{8}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$. [4]

(c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leq x \leq \cos\left(\frac{1}{8}\pi\right)$ and $0 \leq y \leq \sin\left(\frac{1}{8}\pi\right)$. [4]

(d) Using your answer to (b), deduce the exact value of the area bounded by C , the x -axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

- (a) Sketch C .

[2]

(b) Find the exact value of the area of the region enclosed by C and the initial line.

[4]

- [6]

- 5 The curve C has polar equation $r = \theta e^{\frac{1}{8}\theta}$, for $0 \leq \theta \leq 2\pi$.

(a) Sketch C .

[2]

- (b) Find the area of the region bounded by C and the initial line, giving your answer in the form $(p\pi^2 + q\pi + r)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

[6]

7.

(c) Show that, at the point of C furthest from the initial line,

$$\theta \cos \theta + \left(\frac{1}{8}\theta + 1\right) \sin \theta = 0$$

and verify that this equation has a root between 5 and 5.05.

[5]

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7 The curve C has polar equation $r^2 = e^{\sin \theta} \cos \theta$, for $-\frac{1}{2}\pi \leq \theta \leq \frac{1}{2}\pi$.

- (a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

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- (b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

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- (c) Sketch C . [3]

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- (d) Find the area of the region bounded by C , giving your answer in exact form. [3]

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For the answer to any question, the question number must be clearly

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