6 The curve C has polar equation $r = \sin 3\theta$, for $0 \leqslant \theta \leqslant \frac{1}{3}\pi$.

(a) Sketch C and state the equation of the line of symmetry. [3]

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(b) Find the exact value of the area of the region enclosed by C. [4]

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In parts **(c)** and **(d)** you may use the identity $\sin 3\theta \equiv 3\sin\theta - 4\sin^3\theta$.

(c) Find the maximum distance of a point on C from the initial line. [5]

(d) Find a Cartesian equation for C. [3]

5 The curve C has polar equation $r^2 = \tan 2\theta$, where $0 \leqslant \theta \leqslant \frac{1}{8}\pi$.

(a) Sketch C and state the greatest distance of a point on C from the pole. [2]

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(b) Find the exact value of the area of the region bounded by C and the half-line $\theta = \frac{1}{8}\pi$. [4]

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(c) Show that C has Cartesian equation $x^4 - 2xy - y^4 = 0$ given that $0 \leqslant x \leqslant \cos\left(\frac{1}{8}\pi\right)$ and $0 \leqslant y \leqslant \sin\left(\frac{1}{8}\pi\right)$.

[4]

(d) Using your answer to **(b)**, deduce the exact value of the area bounded by C, the x-axis and the line $x = \cos\left(\frac{1}{8}\pi\right)$. [2]

6 The curve C has polar equation $r = \cos\frac{1}{2}\theta$, for $0 \leqslant \theta \leqslant \pi$.

(a) Sketch C. [2]

In parts **(b)** and **(c)** you may use the identities $\sin\theta \equiv 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ and $\cos\theta \equiv 2\cos^2\frac{1}{2}\theta - 1$.

(b) Find the exact value of the area of the region enclosed by C and the initial line. [4]

(c) Find the maximum distance of a point on C from the initial line. Give your answer in the form $p\sqrt{q}$, where p and q are rational numbers to be determined. [6]

5 The curve C has polar equation $r = \theta e^{\frac{1}{8}\theta}$, for $0 \leqslant \theta \leqslant 2\pi$.

(a) Sketch C. [2]

(b) Find the area of the region bounded by C and the initial line, giving your answer in the form $\left(p\pi^2 + q\pi + r\right)e^{\frac{1}{2}\pi} + s$, where p, q, r and s are integers to be determined. [6]

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(c) Show that, at the point of C furthest from the initial line,

$$\theta\cos\theta+\left(\tfrac{1}{8}\theta+1\right)\sin\theta=0$$

and verify that this equation has a root between 5 and 5.05. [5]

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7 The curve C has polar equation $r^2 = e^{\sin\theta}\cos\theta$, for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

(a) Find the polar coordinates of the point on C that is furthest from the pole, giving your answers correct to 3 decimal places. [5]

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(b) Find the polar coordinates of the point on C that is furthest from the half-line $\theta = \frac{1}{2}\pi$, giving your answers correct to 3 decimal places. [5]

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(c) Sketch C. [3]

(d) Find the area of the region bounded by C, giving your answer in exact form. [3]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.