

$$\mathbf{A} = \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 1 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{pmatrix}.$$

- (b) Give full details of the geometrical transformation in the x - y plane represented by $\mathbf{B}$. [2]

(c) Show that the triangles DEF and PQR have the same area. [2]

- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{AB}$. [5]

4 Let k and m be non-zero constants. The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are given by

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} k & 0 \\ 0 & m \end{pmatrix} \text{ and } \mathbf{C} = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \end{pmatrix}.$$

(a) Give full details of the geometrical transformation in the x - y plane represented by the matrix $\mathbf{B}$ in each of the following cases.

(i) $m = 1$ [2]

(ii) $m = k$ [2]

(b) Show that the matrix $\mathbf{ABC}$ is singular. [6]

4 The matrix $\mathbf{M}$ is given by $\mathbf{M} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$, where $0 < \theta < 2\pi$.

- (a) The matrix $\mathbf{M}$ represents a sequence of two geometrical transformations in the x - y plane.

State the type of each transformation, and make clear the order in which they are applied. [2]

[2]

- (b) Find the value of θ for which the transformation represented by $\mathbf{M}$ has a line of invariant points. [7]

- 1** The matrix $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by a stretch parallel to the x -axis, scale factor 14, followed by a rotation anticlockwise about the origin through angle $\frac{1}{3}\pi$.

(a) Show that $2\mathbf{M} = \begin{pmatrix} 14 & -\sqrt{3} \\ 14\sqrt{3} & 1 \end{pmatrix}$. [4]

- (b) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by $\mathbf{M}$. [5]

- The unit square S in the x - y plane is transformed by $\mathbf{M}$ onto the rectangle P .

- (c) Find the matrix which transforms P onto S . [2]