1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}(8r^3+12r^2+4r+3)$ in terms of n, simplifying your answer. [3]

(b) Show that

$$\frac{1}{r^4} - \frac{1}{(r+1)^4} = \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$$

and hence use the method of differences to find $\sum_{r=1}^{n} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{4r^3 + 6r^2 + 4r + 1}{r^4(r+1)^4}$. [1]

1 **(a)** Use standard results from the list of formulae (MF19) to find $\sum_{r=1}^{n}\left(r^3 - r\right)$ in terms of n, fully factorising your answer. [3]

(b) Express $\dfrac{r+3}{r^3-r}$ in the form $\dfrac{A}{r-1}+\dfrac{B}{r}+\dfrac{C}{r+1}$, where A, B and C are constants to be determined, and hence use the method of differences to find $\displaystyle\sum_{r=2}^{n}\frac{r+3}{r^3-r}$. [5]

(c) Deduce the value of $\displaystyle\sum_{r=2}^{\infty}\frac{r+3}{r^3-r}$. [1]

3 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n} (r+1)(r+2)(r+3) = \tfrac{1}{4}n\left(n^3 + bn^2 + cn + d\right),$$

where b, c and d are integers to be determined. [3]

(b) Express $\frac{2}{(r+1)(r+2)(r+3)}$ in partial fractions and hence use the method of differences to find

$$\sum_{r=1}^{n} \frac{2}{(r+1)(r+2)(r+3)}.$$ [5]

(c) State the value of $\sum_{r=1}^{\infty} \frac{2}{(r+1)(r+2)(r+3)}$. [1]

1 **(a)** Use standard results from the list of formulae (MF19) to show that

$$\sum_{r=1}^{n}(2-3r)(5-3r) = an^3 + bn^2 + cn,$$

where a, b and c are integers to be determined. [3]

(b) Use the method of differences to find $\sum_{r=1}^{n} \frac{1}{(2-3r)(5-3r)}$ in terms of n. [4]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1}{(2-3r)(5-3r)}$. [1]

4 Let $w_r = r(r+1)(r+2)\ldots(r+9)$.

(a) Show that

$$w_{r+1} - w_r = 10(r+1)(r+2)\ldots(r+9).$$ [2]

(b) Given that $u_r = (r+1)(r+2)\ldots(r+9)$, find $\sum_{r=1}^{n} u_r$ in terms of n. [3]

(c) Given that $v_r = x^{w_{r+1}} - x^{w_r}$, find the set of values of x for which the infinite series

$$v_1 + v_2 + v_3 + \dots$$

is convergent and give the sum to infinity when this exists. [3]