4(a)	$y=2x+1 \Rightarrow x=\frac{1}{2}(y-1)$
	$\frac{1}{16}(y-1)^4+\frac{1}{8}(y-1)^3+\frac{1}{4}(y-1)^2+\frac{1}{2}(y-1)+1=0$ $\frac{1}{16}(y^4-4y^3+6y^2-4y+1)+\frac{1}{8}(y^3-3y^2+3y-1)+\frac{1}{4}(y^2-2y+1)+\frac{1}{2}(y-1)+1=0$
	$\frac{1}{16}y^4-\frac{1}{8}y^3+\frac{1}{4}y^2+\frac{1}{8}y+\frac{11}{16}=0 \Rightarrow y^4-2y^3+4y^2+2y+11=0$
4(b)	$S_2=2^2-2(4)$
	$S_2=-4$
4(c)	$S_4-2S_3+4S_2+2S_1+44=0$
	$S_4=-76$

2(a)	$b=-(\alpha+\beta+\gamma)=-2$
	$3=(-2)^2-2c$
	$c=\frac{1}{2}$
2(b)	$S_3+bS_2+cS_1+3d=0$
	$S_3=-3b-2c-3d=5-3d$
	$S_4+bS_3+cS_2+dS_1=0$
	$5+(-2)(5-3d)+\left(\frac{1}{2}\right)(3)+2d=0 \Rightarrow 8d-\frac{7}{2}=0$
	$d=\frac{7}{16}$

2(a)	$\alpha=-d$
	$\alpha+\beta+\dfrac{1}{\beta}=-b$
	$\beta+\dfrac{1}{\beta}=d-b$
2(b)	$-d\beta+1-\dfrac{d}{\beta}=c$
	$\beta+\dfrac{1}{\beta}=\dfrac{1-c}{d}$
2(c)(i)	$d-3=\dfrac{1+3}{d} \Rightarrow d^2-3d-4=0$
	$d=4$
2(c)(ii)	$\alpha^2+\beta^2+\gamma^2=b^2-2c$
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A-Level Further Mathematics: **9231 Mathematics - Further June 2025 Question Paper 11**

2(a)	$y = x^3 - 1$
	$(y+1)^{\frac{3}{3}} + 2(y+1)^{\frac{1}{3}} + 1 = 0 \Rightarrow 2(y+1)^{\frac{1}{3}} = -y-2$ $8(y+1) = -(y+2)^3 \Rightarrow 8y + 8 = -y^3 - 6y^2 - 12y - 8$
	$y^3 + 6y^2 + 20y + 16 = 0$
2(b)	$(\alpha^3-1)^2 + (\beta^3-1)^2 + (\gamma^3-1)^2 = 6^2 - 2(20)$
	-4
2(c)	$(\alpha^3-1)^3 + (\beta^3-1)^3 + (\gamma^3-1)^3 = -6(-4) - 20(-6) - 16(3)$
	96

A-Level Further Mathematics: **9231 Mathematics - Further June 2025 Question Paper 13**

3(a)	$0 = \alpha^2 + \beta^2 + \gamma^2 + \delta^2 + 2(7)$
	$\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = -14$
3(b)	$S_4 + 7S_2 + 3S_1 + 88 = 0$
	$S_4 = 10$

er Mathematics: **9231 Mathematics - Further June 2025 Question Paper 13**

3(c)	$(r+\alpha^2)^2 = r^2 + 2\alpha^2 r + \alpha^4$
	$\sum_{r=1}^{10}\left((r+\alpha^2)^2 + (r+\beta^2)^2 + (r+\gamma^2)^2 + (r+\delta^2)^2\right)$ $= \sum_{r=1}^{10}(4r^2 - 28r + 10)$
	$= 4\left(\frac{10}{6}(10+1)(2(10)+1)\right) - 14(10)(10+1)$ $+10(10)$
	$= 100$