

4 The quartic equation $x^4 + x^3 + x^2 + x + 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4 - 2y^3 + 4y^2 + 2y + 11 = 0. \quad [4]$$

The sum $(2\alpha+1)^n + (2\beta+1)^n + (2\gamma+1)^n + (2\delta+1)^n$ is denoted by S_n .

(b) Find the value of S_7 . [2]

(c) Given that $S_3 = -22$, find the value of S_4 . [2]

- [5]

[3]

2 The cubic equation $x^3 + bx^2 + cx + d = 0$, where $d \neq 0$, has roots α, β, γ such that $\gamma = \frac{1}{\beta}$.

(a) Show that $\beta + \frac{1}{\beta} = d - b$. [3]

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(b) Show also that $\beta + \frac{1}{\beta} = \frac{1-c}{d}$. [2]

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(c) It is given that $b = 3, c = -3$ and $d > 0$.

(i) Find the value of d . [2]

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(ii) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

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2 The cubic equation $x^3 + 2x + 1 = 0$ has roots α, β, γ .

(a) Find a cubic equation whose roots are $\alpha^3 - 1, \beta^3 - 1, \gamma^3 - 1$.

[3]

(b) Find the value of $(\alpha^3 - 1)^2 + (\beta^3 - 1)^2 + (\gamma^3 - 1)^2$.

[2]

(c) Find the value of $(\alpha^3 - 1)^3 + (\beta^3 - 1)^3 + (\gamma^3 - 1)^3$.

[2]

3 The quartic equation $x^4 + 7x^2 + 3x + 22 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$. [2]

(b) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

(c) Use standard results from the list of formulae (MF19) to find the value of

$$\sum_{r=1}^{10} \left((\alpha^2 + r)^2 + (\beta^2 + r)^2 + (\gamma^2 + r)^2 + (\delta^2 + r)^2 \right). \quad [5]$$