| | |
|---|---|
| 4(a) | $y=2x+1 \Rightarrow x=\frac{1}{2}(y-1)$ |
| | $\frac{1}{16}(y-1)^4+\frac{1}{8}(y-1)^3+\frac{1}{4}(y-1)^2+\frac{1}{2}(y-1)+1=0$<br>$\frac{1}{16}\left(y^4-4y^3+6y^2-4y+1\right)+\frac{1}{8}\left(y^3-3y^2+3y-1\right)+\frac{1}{4}\left(y^2-2y+1\right)+\frac{1}{2}(y-1)+1=0$ |
| | $\frac{1}{16}y^4-\frac{1}{8}y^3+\frac{1}{4}y^2+\frac{1}{8}y+\frac{11}{16}=0 \Rightarrow y^4-2y^3+4y^2+2y+11=0$ |
| | |
| 4(b) | $S_2=2^2-2(4)$ |
| | $S_2=-4$ |
| | |
| 4(c) | $S_4-2S_3+4S_2+2S_1+44=0$ |
| | $S_4=-76$ |
| | |

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|---|---|
| 7(a) | $x=-\frac{3}{2}-\frac{1}{2}\sqrt{5}, \quad x=-\frac{3}{2}+\frac{1}{2}\sqrt{5}$ |
| | $y=0$ |
| | |
| 7(b) | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x^2+3x+1)(1)-(x+2)(2x+3)}{\left(x^2+3x+1\right)^2}$ |
| | $x^2+4x+5=0$ |
| | Discriminant is $4^2-20=-4<0$ so no real roots. |
| | |





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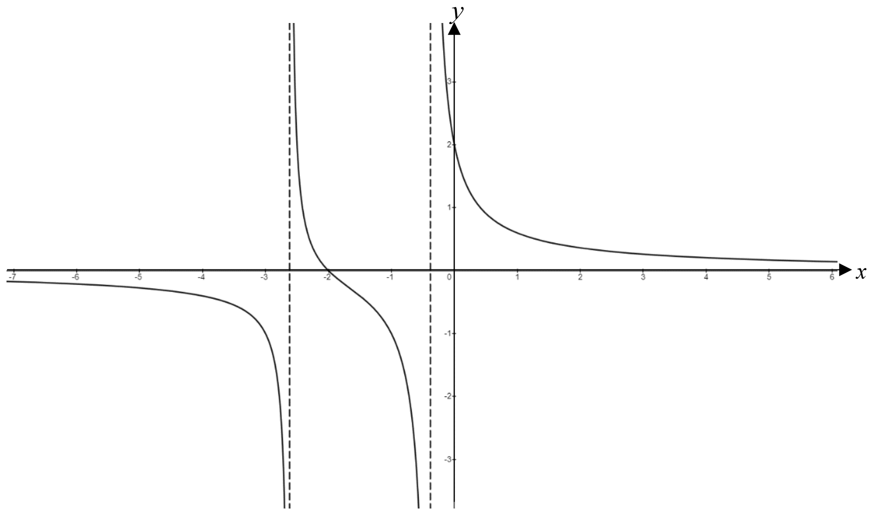
|---|---|
| 7(c) |   |
| | |

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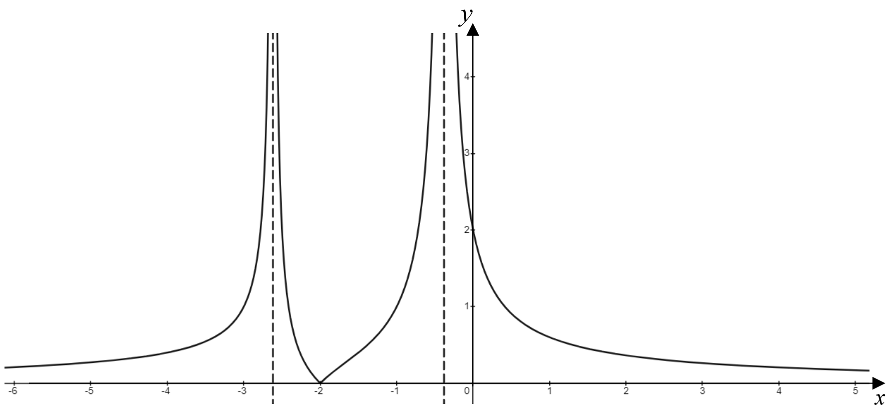
|---|---|
| 7(d) |  |
| | |
| 7(e) | $\frac{x+2}{x^2+3x+1}=2$ or $\frac{x+2}{x^2+3x+1}=-2$<br>$-2x^2-5x=0$ or $2x^2+7x+4=0$ |
| | $x=-\frac{5}{2}$, $x=0$ or $x=-\frac{7}{4}-\frac{1}{4}\sqrt{17}$, $x=-\frac{7}{4}+\frac{1}{4}\sqrt{17}$ |
| | $-\frac{7}{4}-\frac{1}{4}\sqrt{17}<x<-\frac{5}{2}$, $-\frac{7}{4}+\frac{1}{4}\sqrt{17}<x<0$ , $x\neq-\frac{3}{2}\pm\frac{1}{2}\sqrt{5}$ |
| | |



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| 1 | $\begin{vmatrix}1 & -1 & 1\\ 1 & k & 3\\ 1 & 2 & k\end{vmatrix}=\begin{vmatrix}k & 3\\ 2 & k\end{vmatrix}+\begin{vmatrix}1 & 3\\ 1 & k\end{vmatrix}+\begin{vmatrix}1 & k\\ 1 & 2\end{vmatrix}=k^2-7$ |
| | $k=\pm\sqrt{7}$ |
| | |





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| 7(a) | $\frac{\mathrm{d}}{\mathrm{d}x}\left(\tanh^{-1}x\right)=\frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{(1-x)(1)-(1+x)(-1)}{(1-x)^2}$ |
| | $=\frac{1}{2}\left(\frac{1-x}{1+x}\right)\frac{2}{(1-x)^2}=\frac{1}{1-x^2}$ |
| | **Alternative method for question 7(a)** |
| | $\tanh y=x\Rightarrow\operatorname{sech}^2 y\frac{\mathrm{d}y}{\mathrm{d}x}=1$ |
| | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1-\tanh^2 y}=\frac{1}{1-x^2}$ |
| | |
| 7(b) | $\frac{\mathrm{d}y}{\mathrm{d}x}-\frac{y}{x}=x\tanh^{-1}x$ |
| | $\mathrm{e}^{-\int x^{-1}\mathrm{d}\theta}=\mathrm{e}^{-\ln x}=x^{-1}$ |
| | $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^{-1}y\right)=\tanh^{-1}x$ |
| | $x^{-1}y=x\tanh^{-1}x-\int\frac{x}{1-x^2}\,\mathrm{d}x$ |
| | $x^{-1}y=x\tanh^{-1}x+\frac{1}{2}\ln\left(1-x^2\right)+C$ |
| | $0=\frac{1}{2}\tanh^{-1}\frac{1}{2}+\frac{1}{2}\ln\frac{3}{4}+C\Rightarrow C=-\frac{1}{4}\ln 3-\frac{1}{2}\ln\frac{3}{4}$ |
| | $x^{-1}y=x\tanh^{-1}x+\frac{1}{2}\ln\left(1-x^2\right)-\frac{1}{4}\ln\frac{27}{16}$ |
| | |

| | |
|---|---|
| 5(a) | $\overline{x} = ka + \frac{1}{3}h$ |
| | $\overline{x} = \frac{1}{3}(3ka + h)$ |
| | $\overline{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$ |
| | **Alternative method for question 5(a)** |
| | Coordinates $A(ka,0)$, $B(ka+h,h)$, $C(ka,2a)$<br>$\overline{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$ |
| | $\overline{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$ |
| | |





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| 5(b) | Taking moments about the $x$-axis:<br>$\overline{x}\left(2ka^2 + ah\right) = 2ka^2 \times \frac{1}{2}ka + ah\left(ka + \frac{1}{3}h\right)$ |
| | For equilibrium, $\overline{x} \leqslant ka$, so<br>$\dfrac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leqslant ka$ |
| | $(0 \leqslant)\,h \leqslant \sqrt{3}ka$ |
| | |
| 5(c) | Point of toppling means $h = \sqrt{3}ka\,[= a]$.<br>$\overline{x} = \frac{1}{3}a\left(\sqrt{3} + 1\right)$ |
| | [ $h = a$ , meaning triangle $ABC$ is isosceles.]<br>$\overline{y} = a$ (by symmetry) |
| | |

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|---|---|
| 7(a) | $\frac{dy}{dx} = \tan\theta - \frac{2gx}{2u^2\cos^2\theta}$ |
| | $\frac{dy}{dx} = -1$ |
| | $-1 = \frac{4}{3} - \frac{2}{45}x_A$ |
| | $x_A = \frac{105}{2}$ |
| | $y_A = \frac{35}{4}$ |
| | **Alternative method for question 7(a)** |
| | Horizontally: $v_H = 25\cos\theta = 15$<br>Vertically: $v_V = 25\sin\theta - gt = 20 - gt$ |
| | At $A$: $\frac{v_V}{v_H} = -\tan 45° = -1$ |
| | $\frac{20-gt}{15} = -1, \ t = \frac{7}{2}$ |
| | $x_A = \frac{105}{2}$ |
| | $y_A = \frac{35}{4}$ |
| | |





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| 7(b) | Let $v$ be the velocity before the collision at $A$.<br>$u_x = u\cos\theta = 15 = v_x$ |
| | $v_y = -v_x = -15$ |
| | Speed: $\sqrt{v_x^2 + v_y^2} = 15\sqrt{2}$ (m s$^{-1}$) |
| | |
| 7(c) | Let $w$ be the velocity after the collision.<br>$w = ev = \frac{5}{3}\sqrt{2}$ |
| | $-\frac{35}{4} = x\tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2(\cos 45)^2}$ , $\frac{9}{5}x^2 - x - \frac{35}{4} = 0$ .<br>OR<br>$\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, \ t = \frac{3}{2}$<br>$x = (w\cos 45)t \ \left[= \frac{5}{3} \times \frac{3}{2}\right]$ |
| | $x = \frac{5}{2}$ (metres) |
| | |

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|---|---|
| 1(a) | $s^2 = \frac{1}{9}\left('71314' - \frac{'844'^2}{10}\right) \quad \left[= \frac{134}{15} = 8.933\right]$ |
| | 2.262 |
| | $\frac{'844'}{10} \pm '2.262'\sqrt{\frac{'8.933'}{10}}$ |
| | [82.3, 86.5] |
| | The distribution of estimates of angles is normal.<br>OR<br>The estimates are a random sample from some population.<br>OR<br>The estimates are independent.<br>OR<br>Underlying distribution is normal. |
| | |
| 1(b) | Population unlikely to be normal as $\theta$ is close to right-angle / data is skewed.<br>OR<br>Estimates may not be independent, for example due to collusion.<br>OR<br>No indication that sample is random. |
| | |





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|---|---|
| 5(a) | $\frac{1}{16}\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 + \frac{1}{k}\left[2\sqrt{x}\right]_4^9 = 1$<br>$\frac{1}{24}(8-0) + \frac{2}{k}(3-2) = 1$<br>OR<br>$\frac{1}{3} + \frac{2}{k} = 1$ |
| | $\frac{2}{k} = \frac{2}{3}$, $k = 3$ |
| | |
| 5(b) | $\frac{1}{16}\int_0^4 \sqrt{x}\,\mathrm{d}x + \frac{1}{3}\int_4^m x^{-\frac{1}{2}}\,\mathrm{d}x = \frac{1}{2}$<br>OR<br>$\frac{1}{3}\int_m^9 x^{-\frac{1}{2}}\,\mathrm{d}x = 0.5$ |
| | $\frac{1}{3} + \frac{2}{3}\left(\sqrt{m} - 2\right) = \frac{1}{2}$<br>OR<br>$\frac{2}{3}\left(3 - \sqrt{m}\right) = \frac{1}{2}$ |
| | $m = \frac{81}{16} \quad [= 5.0625]$ |
| | |

| | |
|---|---|
| 5(c) | $\mathrm{F}(x)=\begin{cases}\frac{1}{24}x^{\frac{3}{2}} & 0\leqslant x<4\\ \frac{2}{3}x^{\frac{1}{2}}-1 & 4\leqslant x\leqslant 9\end{cases}$ |
| | $\mathrm{G}(y)=\begin{cases}\frac{1}{24}y^{3} & 0\leqslant y<2\\ \frac{2}{3}y-1 & 2\leqslant y\leqslant 3\end{cases}$ |
| | $\mathrm{g}(y)=\begin{cases}\frac{1}{8}y^{2} & 0\leqslant y<2\\ \frac{2}{3} & 2\leqslant y\leqslant 3\\ 0 & \text{otherwise}\end{cases}$ |
| | **Alternative method for question 5(c)** |
| | Using chain rule (or inverse function): $\mathrm{G}(y)=\mathrm{F}\left(y^{2}\right)$ so $\mathrm{g}(y)=\frac{\mathrm{d}}{\mathrm{d}y}\mathrm{F}\left(y^{2}\right)=2y\mathrm{f}\left(y^{2}\right)$. |
| | $\mathrm{g}(y)=\begin{cases}2y\left(\frac{1}{16}\sqrt{y^{2}}\right) & 0\leqslant y<2\\ 2y\left(\dfrac{1}{3\sqrt{y^{2}}}\right) & 2\leqslant y\leqslant 3\end{cases}$ |
| | $\mathrm{g}(y)=\begin{cases}\frac{1}{8}y^{2} & 0\leqslant y<2\\ \frac{2}{3} & 2\leqslant y\leqslant 3\\ 0 & \text{otherwise}\end{cases}$ |
| | |