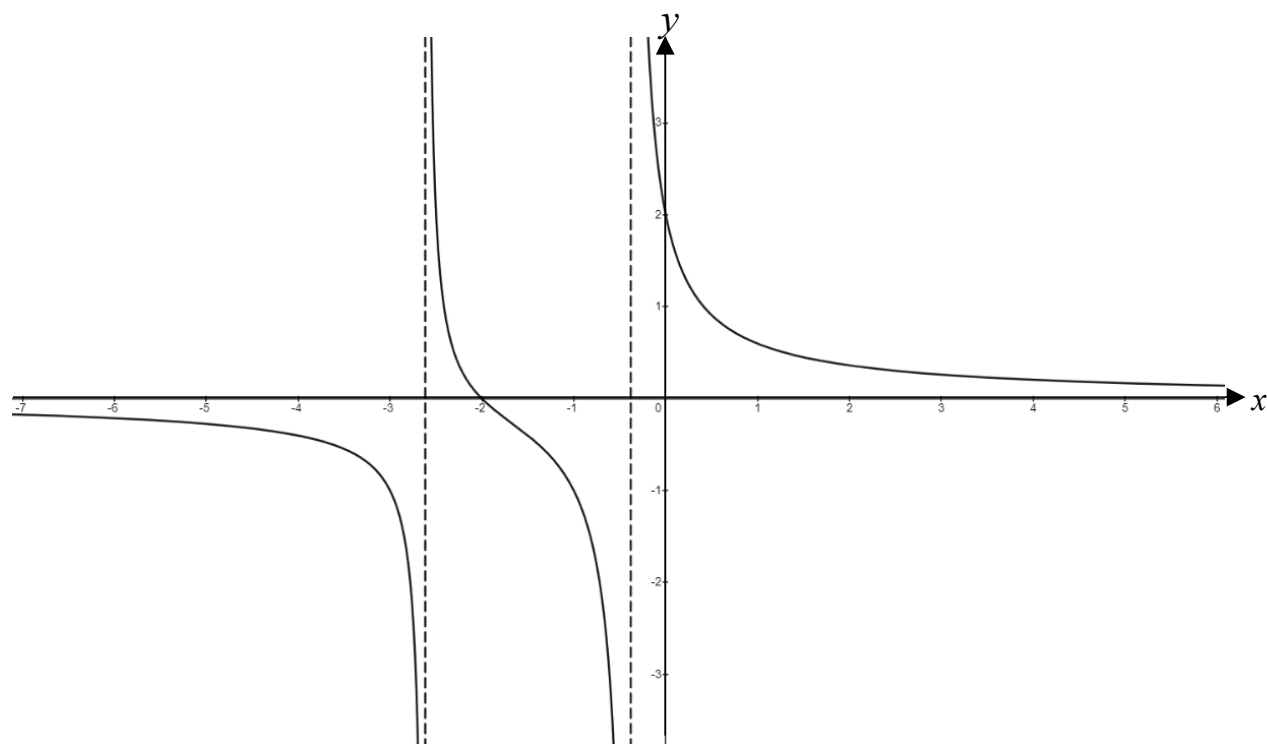
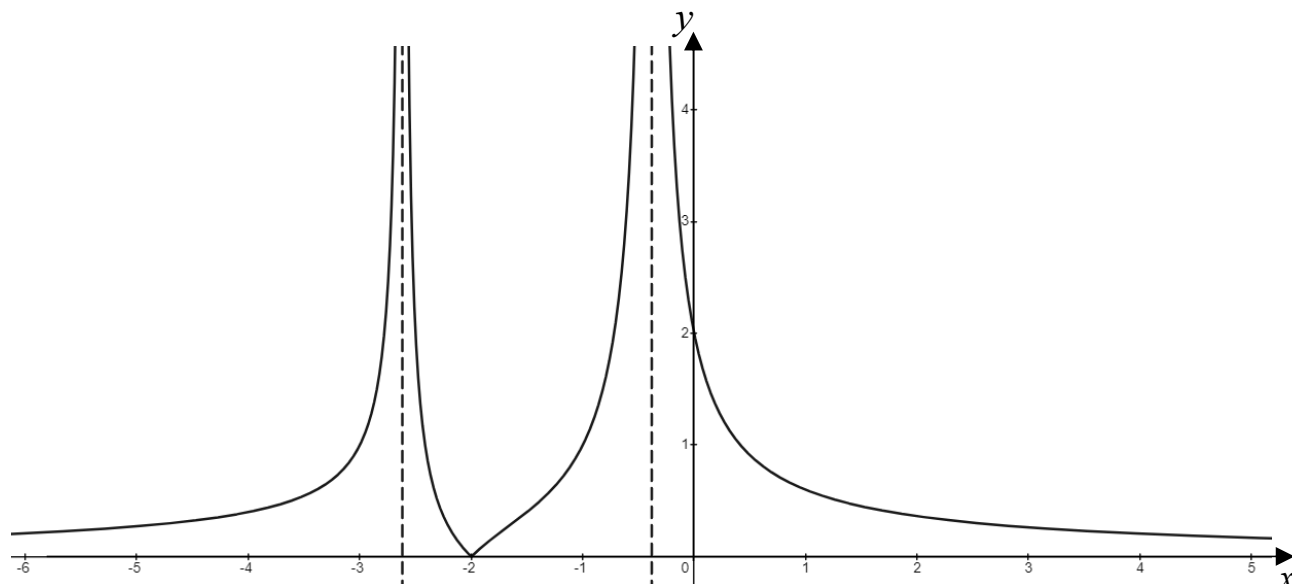


4(a)	$y = 2x + 1 \Rightarrow x = \frac{1}{2}(y - 1)$
	$\frac{1}{16}(y-1)^4 + \frac{1}{8}(y-1)^3 + \frac{1}{4}(y-1)^2 + \frac{1}{2}(y-1) + 1 = 0$ $\frac{1}{16}(y^4 - 4y^3 + 6y^2 - 4y + 1) + \frac{1}{8}(y^3 - 3y^2 + 3y - 1) + \frac{1}{4}(y^2 - 2y + 1) + \frac{1}{2}(y - 1) + 1 = 0$
	$\frac{1}{16}y^4 - \frac{1}{8}y^3 + \frac{1}{4}y^2 + \frac{1}{8}y + \frac{11}{16} = 0 \Rightarrow y^4 - 2y^3 + 4y^2 + 2y + 11 = 0$
4(b)	$S_2 = 2^2 - 2(4)$
	$S_2 = -4$
4(c)	$S_4 - 2S_3 + 4S_2 + 2S_1 + 44 = 0$
	$S_4 = -76$
7(a)	$x = -\frac{3}{2} - \frac{1}{2}\sqrt{5}, \quad x = -\frac{3}{2} + \frac{1}{2}\sqrt{5}$
	$y = 0$
7(b)	$\frac{dy}{dx} = \frac{(x^2 + 3x + 1)(1) - (x + 2)(2x + 3)}{(x^2 + 3x + 1)^2}$
	$x^2 + 4x + 5 = 0$
	Discriminant is $4^2 - 20 = -4 < 0$ so no real roots.

7(c)



7(d)



7(e)

$$\frac{x+2}{x^2+3x+1} = 2 \text{ or } \frac{x+2}{x^2+3x+1} = -2$$

$$-2x^2 - 5x = 0 \text{ or } 2x^2 + 7x + 4 = 0$$

$$x = -\frac{5}{2}, \quad x = 0 \text{ or } x = -\frac{7}{4} - \frac{1}{4}\sqrt{17}, \quad x = -\frac{7}{4} + \frac{1}{4}\sqrt{17}$$

$$-\frac{7}{4} - \frac{1}{4}\sqrt{17} < x < -\frac{5}{2}, \quad -\frac{7}{4} + \frac{1}{4}\sqrt{17} < x < 0, \quad x \neq -\frac{3}{2} \pm \frac{1}{2}\sqrt{5}$$

1

$$\begin{vmatrix} 1 & -1 & 1 \\ 1 & k & 3 \\ 1 & 2 & k \end{vmatrix} = \begin{vmatrix} k & 3 \\ 2 & k \end{vmatrix} + \begin{vmatrix} 1 & 3 \\ 1 & k \end{vmatrix} + \begin{vmatrix} 1 & k \\ 1 & 2 \end{vmatrix} = k^2 - 7$$

$$k = \pm\sqrt{7}$$

7(a)	$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{(1-x)(1) - (1+x)(-1)}{(1-x)^2}$
	$= \frac{1}{2} \left(\frac{1-x}{1+x} \right) \frac{2}{(1-x)^2} = \frac{1}{1-x^2}$
	Alternative method for question 7(a)
	$\tanh y = x \Rightarrow \operatorname{sech}^2 y \frac{dy}{dx} = 1$
	$\frac{dy}{dx} = \frac{1}{1 - \tanh^2 y} = \frac{1}{1 - x^2}$
7(b)	$\frac{dy}{dx} - \frac{y}{x} = x \tanh^{-1} x$
	$e^{-\int x^{-1} d\theta} = e^{-\ln x} = x^{-1}$
	$\frac{d}{dx}(x^{-1}y) = \tanh^{-1} x$
	$x^{-1}y = x \tanh^{-1} x - \int \frac{x}{1-x^2} dx$
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) + C$
	$0 = \frac{1}{2} \tanh^{-1} \frac{1}{2} + \frac{1}{2} \ln \frac{3}{4} + C \Rightarrow C = -\frac{1}{4} \ln 3 - \frac{1}{2} \ln \frac{3}{4}$
	$x^{-1}y = x \tanh^{-1} x + \frac{1}{2} \ln(1-x^2) - \frac{1}{4} \ln \frac{27}{16}$

5(a)

$$\bar{x} = ka + \frac{1}{3}h$$

$$\bar{x} = \frac{1}{3}(3ka + h)$$

$$\bar{y} = a + \frac{1}{3}(h - a) = \frac{1}{3}(2a + h)$$

Alternative method for question 5(a)

Coordinates $A(ka, 0)$, $B(ka + h, h)$, $C(ka, 2a)$

$$\bar{x} = \frac{1}{3}(ka + ka + h + ka) = \frac{1}{3}(3ka + h)$$

$$\bar{y} = \frac{1}{3}(0 + h + 2a) = \frac{1}{3}(h + 2a)$$

5(b)

Taking moments about the x -axis:

$$\bar{x}(2ka^2 + ah) = 2ka^2 \times \frac{1}{2}ka + ah\left(ka + \frac{1}{3}h\right)$$

For equilibrium, $\bar{x} \leq ka$, so

$$\frac{a^2k^2 + ahk + \frac{1}{3}h^2}{2ak + h} \leq ka$$

$$(0 \leq) h \leq \sqrt{3}ka$$

5(c)

Point of toppling means $h = \sqrt{3}ka [= a]$.

$$\bar{x} = \frac{1}{3}a(\sqrt{3} + 1)$$

[$h = a$, meaning triangle ABC is isosceles.]

$$\bar{y} = a \text{ (by symmetry)}$$

7(a)

$$\frac{dy}{dx} = \tan \theta - \frac{2gx}{2u^2 \cos^2 \theta}$$

$$\frac{dy}{dx} = -1$$

$$-1 = \frac{4}{3} - \frac{2}{45}x_A$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

Alternative method for question 7(a)

Horizontally: $v_H = 25 \cos \theta = 15$

Vertically: $v_V = 25 \sin \theta - gt = 20 - gt$

At A: $\frac{v_V}{v_H} = -\tan 45^\circ = -1$

$$\frac{20 - gt}{15} = -1, \quad t = \frac{7}{2}$$

$$x_A = \frac{105}{2}$$

$$y_A = \frac{35}{4}$$

7(b)

Let v be the velocity before the collision at A .

$$u_x = u \cos \theta = 15 = v_x$$

$$v_y = -v_x = -15$$

$$\text{Speed: } \sqrt{v_x^2 + v_y^2} = 15\sqrt{2} \text{ (ms}^{-1}\text{)}$$

7(c)

Let w be the velocity after the collision.

$$w = ev = \frac{5}{3}\sqrt{2}$$

$$-\frac{35}{4} = x \tan 45 - \frac{gx^2}{2\left(\frac{5}{3}\sqrt{2}\right)^2 (\cos 45)^2}, \quad \frac{9}{5}x^2 - x - \frac{35}{4} = 0.$$

OR

$$\frac{35}{4} = -\frac{5}{3}\sqrt{2} \times \frac{1}{2}\sqrt{2}t + \frac{1}{2}gt^2, \quad t = \frac{3}{2}$$

$$x = (w \cos 45)t \quad \left[= \frac{5}{3} \times \frac{3}{2} \right]$$

$$x = \frac{5}{2} \text{ (metres)}$$

1(a)	$s^2 = \frac{1}{9} \left('71314' - \frac{'844'^2}{10} \right) \quad \left[= \frac{134}{15} = 8.933 \right]$
	2.262
	$\frac{'844'}{10} \pm '2.262' \sqrt{\frac{'8.933'}{10}}$
	[82.3, 86.5]
	The distribution of estimates of angles is normal. OR The estimates are a random sample from some population. OR The estimates are independent. OR Underlying distribution is normal.
1(b)	Population unlikely to be normal as θ is close to right-angle / data is skewed. OR Estimates may not be independent, for example due to collusion. OR No indication that sample is random.

5(a)	$\frac{1}{16} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4 + \frac{1}{k} \left[2\sqrt{x} \right]_4^9 = 1$ $\frac{1}{24}(8-0) + \frac{2}{k}(3-2) = 1$ OR $\frac{1}{3} + \frac{2}{k} = 1$
	$\frac{2}{k} = \frac{2}{3}, k = 3$
5(b)	$\frac{1}{16} \int_0^4 \sqrt{x} \, dx + \frac{1}{3} \int_4^m x^{-\frac{1}{2}} \, dx = \frac{1}{2}$ OR $\frac{1}{3} \int_m^9 x^{-\frac{1}{2}} \, dx = 0.5$
	$\frac{1}{3} + \frac{2}{3}(\sqrt{m} - 2) = \frac{1}{2}$ OR $\frac{2}{3}(3 - \sqrt{m}) = \frac{1}{2}$
	$m = \frac{81}{16} \quad [= 5.0625]$

5(c)

$$F(x) = \begin{cases} \frac{1}{24}x^{\frac{3}{2}} & 0 \leq x < 4 \\ \frac{2}{3}x^{\frac{1}{2}} - 1 & 4 \leq x \leq 9 \end{cases}$$

$$G(y) = \begin{cases} \frac{1}{24}y^3 & 0 \leq y < 2 \\ \frac{2}{3}y - 1 & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Alternative method for question 5(c)

Using chain rule (or inverse function):

$$G(y) = F(y^2) \text{ so } g(y) = \frac{d}{dy} F(y^2) = 2yf(y^2).$$

$$g(y) = \begin{cases} 2y\left(\frac{1}{16}\sqrt{y^2}\right) & 0 \leq y < 2 \\ 2y\left(\frac{1}{3\sqrt{y^2}}\right) & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$