4 The quartic equation $x^4+x^3+x^2+x+1=0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Show that a quartic equation with roots $2\alpha+1, 2\beta+1, 2\gamma+1, 2\delta+1$ is

$$y^4-2y^3+4y^2+2y+11=0.$$ [4]

The sum $(2\alpha+1)^n+(2\beta+1)^n+(2\gamma+1)^n+(2\delta+1)^n$ is denoted by S_n.

(b) Find the value of S_2. [2]

(c) Given that $S_3=-22$, find the value of S_4. [2]

7 The curve C has equation $y = \dfrac{x+2}{x^2+3x+1}$.

(a) Find the equations of the asymptotes of C. [2]

(b) Show that C has no stationary points. [3]

(c) Sketch C, stating the coordinates of the intersections with the axes. [3]

(d) Sketch the curve with equation $y = \left|\dfrac{x+2}{x^2+3x+1}\right|$. [2]

(e) Find in exact form the set of values of x for which $\left|\dfrac{x+2}{x^2+3x+1}\right| > 2$. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

1 Find the values of k for which the system of equations

$$\begin{aligned} x - y + z &= 0, \\ x + ky + 3z &= 0, \\ x + 2y + kz &= 0, \end{aligned}$$

does not have a unique solution. [3]

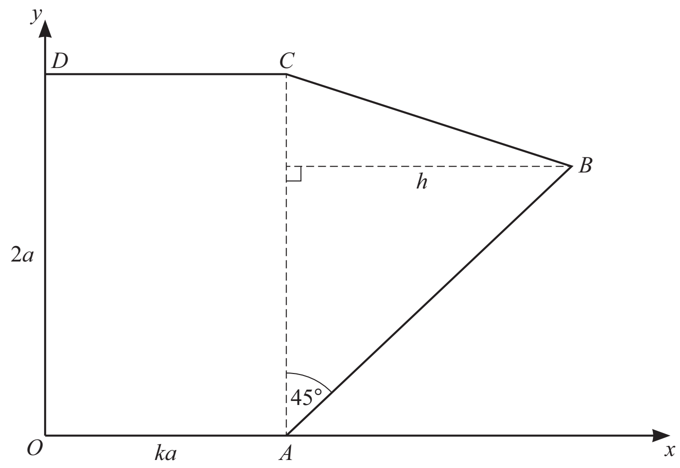
7 **(a)** Show that $\frac{d}{dx}(\tanh^{-1}x) = \frac{1}{1-x^2}$. [3]

(b) Find the solution of the differential equation

$$x\frac{dy}{dx} - y = x^2 \tanh^{-1} x,$$

for $0 < x < 1$, given that $y = 0$ when $x = \frac{1}{2}$. Give your answer in an exact form. [9]

5



A uniform lamina $OABCD$ consists of a rectangle $OACD$ and a triangle ABC. The length of OA is ka, the length of OD is $2a$, the height of triangle ABC is h and angle CAB is $45°$ (see diagram). Relative to axes through O, parallel and perpendicular to OA as shown, the centre of mass of triangle ABC is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\overline{y}$. [3]

The lamina $OABCD$ is placed vertically on its edge OA on a horizontal plane.

(b) Find, in terms of a and k, the set of values of h for which the lamina is in equilibrium. [4]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

(c) Find, in terms of a, the coordinates of the centre of mass of the triangle ABC. [2]

7 A particle P is projected from a point O on a horizontal plane and moves freely under gravity. The initial velocity of P is $25\,\mathrm{m\,s^{-1}}$ at an angle θ above the horizontal, where $\tan\theta = \frac{4}{3}$. At point A, the direction of motion of P makes an angle of $45°$ with the downward vertical through A.

(a) By differentiating the equation of the trajectory or otherwise, find the coordinates of A. [5]

At point A, the particle strikes a fixed smooth barrier, rebounds, and lands on the horizontal plane. The barrier is inclined at an angle of 45° to the horizontal.

(b) Find the speed of P immediately before it collides with the barrier. [3]

(c) Given that the coefficient of restitution between the barrier and the particle is $\frac{1}{9}$, find the horizontal distance travelled by P after it strikes the barrier. [4]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

1 A group of 10 school children are asked to estimate the size of an angle $\theta°$ in a given acute angled triangle. These estimates, in degrees, are as follows.

84 85 77 85 84 87 86 88 83 85

(a) Stating any assumptions you make, calculate a 95% confidence interval for θ. [5]

(b) Give a reason why the assumptions made in part **(a)** may not be appropriate in this case. [1]

5 A continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{16}\sqrt{x} & 0 \leqslant x < 4, \\ \frac{1}{k\sqrt{x}} & 4 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Show that $k = 3$. [2]

(b) Find the median value of X. [3]

The random variable Y is defined by $Y = \sqrt{X}$.

(c) Find the probability density function of Y. [5]