

5(a)	$\frac{1}{16} \left[\frac{2}{3} x^{\frac{3}{2}} \right]_0^4 + \frac{1}{k} \left[2\sqrt{x} \right]_4^9 = 1$ $\frac{1}{24}(8-0) + \frac{2}{k}(3-2) = 1$ OR $\frac{1}{3} + \frac{2}{k} = 1$
	$\frac{2}{k} = \frac{2}{3}, k = 3$
5(b)	$\frac{1}{16} \int_0^4 \sqrt{x} \, dx + \frac{1}{3} \int_4^m x^{-\frac{1}{2}} \, dx = \frac{1}{2}$ OR $\frac{1}{3} \int_m^9 x^{-\frac{1}{2}} \, dx = 0.5$
	$\frac{1}{3} + \frac{2}{3}(\sqrt{m} - 2) = \frac{1}{2}$ OR $\frac{2}{3}(3 - \sqrt{m}) = \frac{1}{2}$
	$m = \frac{81}{16} \quad [= 5.0625]$

5(c)

$$F(x) = \begin{cases} \frac{1}{24}x^{\frac{3}{2}} & 0 \leq x < 4 \\ \frac{2}{3}x^{\frac{1}{2}} - 1 & 4 \leq x \leq 9 \end{cases}$$

$$G(y) = \begin{cases} \frac{1}{24}y^3 & 0 \leq y < 2 \\ \frac{2}{3}y - 1 & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Alternative method for question 5(c)

Using chain rule (or inverse function):

$$G(y) = F(y^2) \text{ so } g(y) = \frac{d}{dy} F(y^2) = 2yf(y^2).$$

$$g(y) = \begin{cases} 2y\left(\frac{1}{16}\sqrt{y^2}\right) & 0 \leq y < 2 \\ 2y\left(\frac{1}{3\sqrt{y^2}}\right) & 2 \leq y \leq 3 \end{cases}$$

$$g(y) = \begin{cases} \frac{1}{8}y^2 & 0 \leq y < 2 \\ \frac{2}{3} & 2 \leq y \leq 3 \\ 0 & \text{otherwise} \end{cases}$$