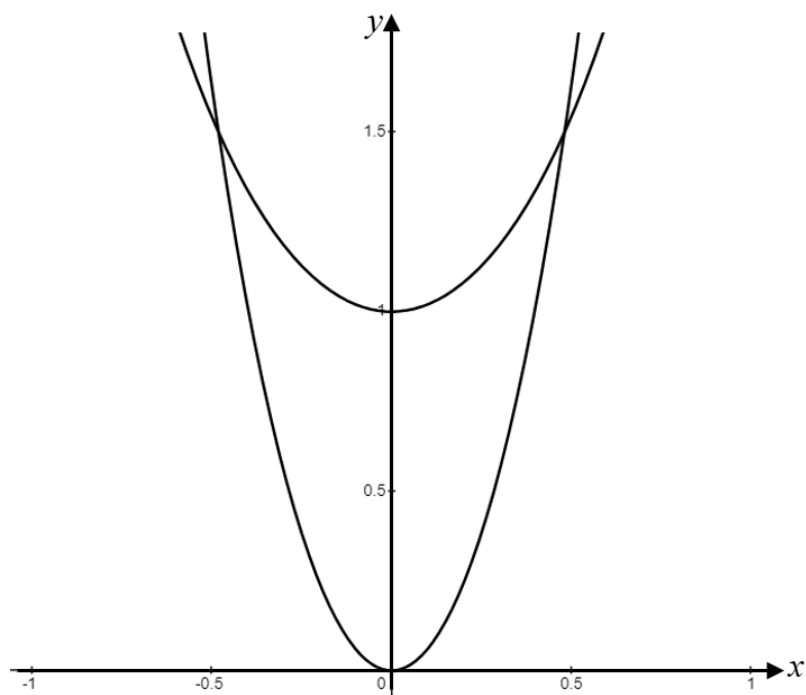


1(b)	$I_2 = \cosh 1 - 2 \sinh 1 + 2I_0$
	$I_2 = 3 \cosh 1 - 2 \sinh 1 - 2$
2(a)	$1 + 2 \sinh^2 x = 6 \sinh^2 x \Rightarrow \sinh x = \pm \frac{1}{2}$
	$x = \sinh^{-1} \left(\pm \frac{1}{2} \right) = \ln \left(\pm \frac{1}{2} + \sqrt{\frac{1}{4} + 1} \right)$
	$x = \ln \left(\pm \frac{1}{2} + \frac{1}{2} \sqrt{5} \right)$

2(b)



3(a)	$\sin 5\theta = \operatorname{Im}(\cos \theta + i \sin \theta)^5 = \sin^5 \theta - 10 \sin^3 \theta \cos^2 \theta + 5 \sin \theta \cos^4 \theta$
	$= \sin^5 \theta - 10 \sin^3 \theta (1 - \sin^2 \theta) + 5 \sin \theta (1 - \sin^2 \theta)^2$
	$= 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$
3(b)	$x = \sin \theta, \quad \sin 5\theta = \frac{1}{2} \sqrt{3}$
	$5\theta = \frac{1}{3} \pi + 2k\pi \quad \text{or} \quad 5\theta = \frac{2}{3} \pi + 2k\pi$
	$\sin\left(\frac{1}{15} \pi\right)$
	$\sin\left(\frac{2}{15} \pi\right), \sin\left(-\frac{5}{15} \pi\right), \sin\left(-\frac{4}{15} \pi\right), \sin\left(\frac{7}{15} \pi\right)$

4(a)	$\int_0^1 \frac{1}{2} (3^x) dx < \frac{1}{2} \left(\left(\frac{1}{N} \right) 3^{\frac{1}{N}} + \left(\frac{1}{N} \right) 3^{\frac{2}{N}} + \dots + \left(\frac{1}{N} \right) 3^{\frac{N-1}{N}} + \left(\frac{1}{N} \right) 3^{\frac{N}{N}} \right)$
	$= \frac{1}{2N} \sum_{n=1}^N \left(3^{\frac{1}{N}} \right)^n = \frac{3^{\frac{1}{N}}}{N \left(3^{\frac{1}{N}} - 1 \right)}$
4(b)	$\int_0^1 \frac{1}{2} (3^x) dx > \frac{1}{2} \left(\left(\frac{1}{N} \right) + \left(\frac{1}{N} \right) 3^{\frac{1}{N}} + \dots + \left(\frac{1}{N} \right) 3^{\frac{N-2}{N}} + \left(\frac{1}{N} \right) 3^{\frac{N-1}{N}} \right)$
	$= \frac{1}{2N} \sum_{n=0}^{N-1} \left(3^{\frac{1}{N}} \right)^n = \frac{1}{N \left(3^{\frac{1}{N}} - 1 \right)}$
4(c)	$\frac{3^{\frac{1}{N}}}{N \left(3^{\frac{1}{N}} - 1 \right)} - \frac{1}{N \left(3^{\frac{1}{N}} - 1 \right)} = \frac{1}{N}$
	$\frac{1}{N} \rightarrow 0 \text{ as } N \rightarrow \infty.$

5(a)	$m^2 + m + 1 = 0 \Rightarrow m = -\frac{1}{2} \pm \frac{1}{2}\sqrt{3}$
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right)$
	$y = p \sin 2x + q \cos 2x \Rightarrow y' = 2p \cos 2x - 2q \sin 2x \Rightarrow y'' = -4p \sin 2x - 4q \cos 2x$
	$-4p - 2q + p = 1 \qquad -4q + 2p + q = 1$
	$p = -\frac{1}{13} \qquad q = -\frac{5}{13}$
	$y = e^{-\frac{1}{2}x} \left(A \cos \frac{\sqrt{3}}{2}x + B \sin \frac{\sqrt{3}}{2}x \right) - \frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x$
5(b)	$y \approx -\frac{1}{13} \sin 2x - \frac{5}{13} \cos 2x = \sqrt{\frac{2}{13}} \left(-\frac{1}{\sqrt{26}} \sin 2x - \frac{5}{\sqrt{26}} \cos 2x \right)$
	$\sqrt{\frac{2}{13}} \sin(2x - 1.77)$

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$$\int \frac{1+x}{1-2x-x^2} dx = -\frac{1}{2} \ln(1-2x-x^2)$$

$$e^{-\frac{1}{2} \ln(1-2x-x^2)} = \frac{1}{\sqrt{1-2x-x^2}}$$

$$\frac{d}{dx} \left(\frac{y}{\sqrt{1-2x-x^2}} \right) = \frac{1}{\sqrt{1-2x-x^2}}$$

$$1-2x-x^2 = 2-(x+1)^2$$

$$\int \frac{1}{\sqrt{2-(x+1)^2}} dx = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + C$$

$$\pi = \sin^{-1} \frac{1}{\sqrt{2}} + C \Rightarrow C = \frac{3}{4} \pi$$

$$\frac{y}{\sqrt{1-2x-x^2}} = \sin^{-1} \left(\frac{x+1}{\sqrt{2}} \right) + \frac{3}{4} \pi$$

7(a)(i)	$\dot{x} = 4 \frac{\sec^2 \frac{1}{2}t}{\tan \frac{1}{2}t} + 3 \operatorname{cosec}^2 t - 3$ $\dot{x} = 8 \operatorname{cosec} t + 3 \operatorname{cosec}^2 t - 3 = 8 \operatorname{cosec} t + 3 \cot^2 t$
7(a)(ii)	$\dot{y} = 6 \operatorname{cosec} t - 4 \cot^2 t$
7(a)(iii)	$\dot{x}^2 + \dot{y}^2 = (8 \operatorname{cosec} t + 3 \cot^2 t)^2 + (6 \operatorname{cosec} t - 4 \cot^2 t)^2$ $= 100 \operatorname{cosec}^2 t + 25 \cot^4 t = 100 \operatorname{cosec}^2 t + 25 (\operatorname{cosec}^2 t - 1)^2$ $= 25 \operatorname{cosec}^4 t + 50 \operatorname{cosec}^2 t + 25$
	$= 25 (\operatorname{cosec}^2 t + 1)^2$
	$5 \int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec}^2 t + 1 \, dt = 5 \left[-\cot t + t \right]_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} = 5 \left(-\frac{1}{\sqrt{3}} + \frac{1}{3}\pi + \sqrt{3} - \frac{1}{6}\pi \right) = \frac{5}{6}\pi + \frac{10}{3}\sqrt{3}$

7(b)	$\frac{\dot{y}}{\dot{x}} = \frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} = \frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} = \frac{6 \sec t - 4 \cot t}{8 \sec t + 3 \cot t}$
	$\frac{d}{dt} \left(\frac{6 \operatorname{cosec} t - 4 \cot^2 t}{8 \operatorname{cosec} t + 3 \cot^2 t} \right)$ $= \frac{(8 \csc t + 3 \cot^2 t)(-6 \cot t \csc t + 8 \cot t \csc^2 t) + (6 \csc t - 4 \cot^2 t)(6 \csc^2 t \cot t + 8 \cot t \csc t)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2}$
	or $\frac{d}{dt} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right)$ $= \frac{(8 \sin t + 3 \cos^2 t)(6 \cos t + 8 \cos t \sin t) - (6 \sin t - 4 \cos^2 t)(8 \cos t - 6 \cos t \sin t)}{(8 \sin t + 3 \cos^2 t)^2}$
	$\frac{100 \cos t \sin^2 t + 50 \cos^3 t}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^2} = \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^2}$
	$\frac{d^2 y}{dx^2} = \frac{d}{dt} \left(\frac{6 \sin t - 4 \cos^2 t}{8 \sin t + 3 \cos^2 t} \right) \times \frac{dt}{dx} = \frac{50 \sin^2 t \cos t (\sin^2 t + 1)}{(8 \sin t + 3 \cos^2 t)^3}$ $= \frac{50 \cot t \operatorname{cosec} t (\operatorname{cosec}^2 t + 1)}{(8 \operatorname{cosec} t + 3 \cot^2 t)^3}$

8(a)	$\begin{vmatrix} 5 & a & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{vmatrix} = -20a - 25$
	$a \neq -\frac{5}{4}$
	Three planes intersect at a <i>single</i> point.