

4 Further Probability & Statistics – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
probability density function	概率密度函数	_____
cumulative distribution function	累积分布函数	_____
median	中位数	_____

Recall

In AS Statistics you used discrete random variables and the normal distribution. Bring this with you:

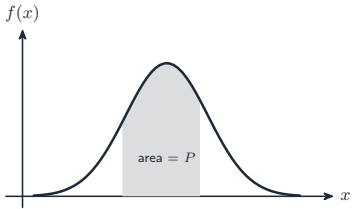
- a discrete variable takes separate values, each with a probability;
- the normal distribution is a bell-shaped curve.

Further Statistics adds **continuous** random variables, described by a curve.

New ideas

■ 1 The probability density function

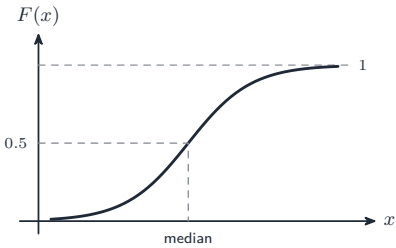
A continuous variable X is described by a **probability density function** 概率密度函数 $f(x)$. The probability that X lies in a range is the **area** under f over that range — and the **total** area under f is 1.



Micro-check. What does the **total** area under a probability density function equal? [1]

■ 2 The cumulative distribution function

The **cumulative distribution function** 累积分布函数 $F(x) = P(X \leq x)$ is the running total — the area to the **left** of x . It rises from 0 to 1. The **median** 中位数 m is the value where the area is split in half: $F(m) = 0.5$.



Worked example. If $f(x) = \frac{1}{2}x$ for $0 \leq x \leq 2$, then $F(x) = \frac{1}{4}x^2$. Setting $F(m) = 0.5$ gives $m^2 = 2$, so $m = \sqrt{2} = 1.41$.

Micro-check. The median m of a continuous variable satisfies $F(m) = ?$ [1]

Watch out: $f(x)$ is a **density**, not a probability — a probability is the **area** under it (an integral), and the total area is 1. The cumulative function $F(x)$ only ever **rises**, from 0 up to 1.

Practice

4.1 State what the **area** under a probability density function over a range represents.^[1]

4.2 State the value of the **total** area under a probability density function. ^[1]

4.3 State what $F(x) = P(X \leq x)$ is called, and what value it rises **to**. ^[2]

4.4 A continuous variable has $F(x) = \frac{1}{4}x^2$ for $0 \leq x \leq 2$. Find the median (solve $F(m) = 0.5$). ^[2]

4.5 State the equation the median m must satisfy. ^[1]

4.6 Challenge. A continuous variable has $f(x) = kx$ for $0 \leq x \leq 3$ (and 0 elsewhere). Find the value of k . ^[3]