

2 De Moivre and Maclaurin – Part 2

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)
De Moivre's theorem	棣莫弗定理	_____
roots of unity	单位根	_____
Maclaurin series	麦克劳林级数	_____

Recall

Further Pure 2 also adds de Moivre's theorem for complex numbers and the Maclaurin series.

New ideas

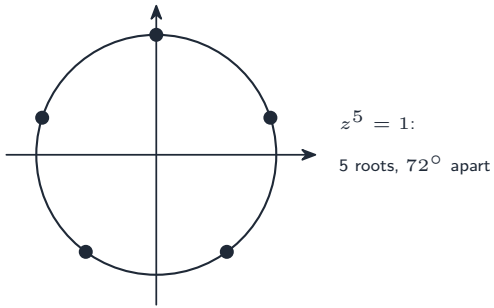
Read these first, then do the Practice.

■ 1 De Moivre's theorem

De Moivre's theorem 棣莫弗定理 says that for any integer n :

$$(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$$

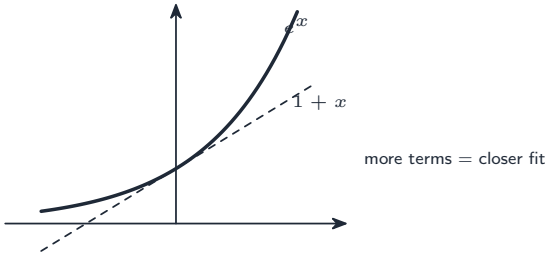
It gives the n **roots of unity** 单位根 — the solutions of $z^n = 1$, equally spaced on the unit circle:



Worked example. With $\theta = 30^\circ$ and $n = 3$: $(\cos 30^\circ + i \sin 30^\circ)^3 = \cos 90^\circ + i \sin 90^\circ = i$.

■ 2 Maclaurin's series

A **Maclaurin series** 麦克劳林级数 writes a function as a power series. Adding more terms fits the curve better:



$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

Watch out: de Moivre multiplies the **angle** by n — it does not raise the angle to a power; the n roots of unity are equally spaced $\frac{360^\circ}{n}$ apart. A Maclaurin series is only a close fit **near** $x = 0$.

Practice

Try these in order.

2.6 Write down de Moivre's theorem. [1]

2.7 How many solutions does $z^5 = 1$ have? [1]

2.8 How far apart, in degrees, are the 5 roots of unity on the circle? [1]

2.9 Write the first three terms of the Maclaurin series for e^x . [1]

2.10 What happens as you add more terms to a Maclaurin series? [1]

2.11 Challenge. Use de Moivre's theorem to simplify $(\cos 20^\circ + i \sin 20^\circ)^9$. [3]