2 Hyperbolic functions and eigenvalues – Part 1

Name: _____ Class: _____ Date: _____

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
hyperbolic functions	双曲函数			
eigenvector	特征向量			
eigenvalue	特征值			
characteristic equation	特征方程			

Recall

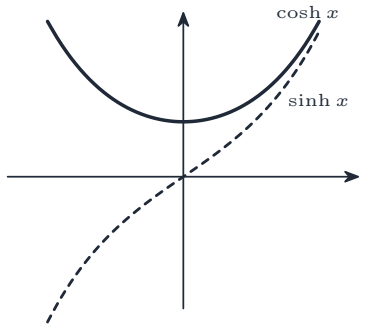
Further Pure 2 adds hyperbolic functions and the eigenvalues of a matrix.

New ideas

Read these first, then do the Practice.

■ 1 Hyperbolic functions

The **hyperbolic functions** 双曲函数 are built from e^x:

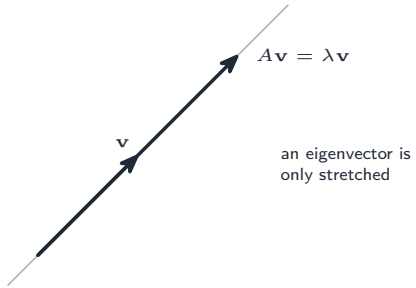


$$\cosh x = \frac{e^x + e^{-x}}{2}, \quad \sinh x = \frac{e^x - e^{-x}}{2}$$

The main identity is $\cosh^2 x - \sinh^2 x = 1$.

■ 2 Eigenvalues

For a square matrix A, an **eigenvector** 特征向量 $\mathbf{v}$ is only stretched (not turned) by A: $A\mathbf{v} = \lambda\mathbf{v}$. The number λ is an **eigenvalue** 特征值:



You find λ from the **characteristic equation** 特征方程 $\det(A - \lambda I) = 0$.

Worked example. For $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$: $\det(A - \lambda I) = (3 - \lambda)(2 - \lambda) = 0$, so the eigenvalues are $\lambda = 3$ and $\lambda = 2$.

Watch out: the hyperbolic identity has a **minus** sign — $\cosh^2 x - \sinh^2 x = 1$ (unlike $\cos^2 x + \sin^2 x = 1$), and $\cosh x \geqslant 1$ always. For eigenvalues, subtract λ from the **diagonal** of A before taking the determinant.

Practice

Try these in order.

2.1 Write $\cosh x$ in terms of e^x. [1]

2.2 What is the main hyperbolic identity? [1]

2.3 Find $\cosh 0$. [1]

2.4 What does it mean for $\mathbf{v}$ to be an *eigenvector* of A? [1]

2.5 Which equation gives the eigenvalues of A? [1]

2.6 Challenge. Find the eigenvalues of $A = \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}$. [3]