1 Further Pure Mathematics 1 – Part 1

Name: ______________ Class: __________ Date: ____________

Vocabulary 词汇

Write each word three times. 把每个单词抄写三遍。

English	中文	Write it 3 times (English)		
roots	根			
coefficients	系数			
matrix	矩阵			
determinant	行列式			
inverse matrix	逆矩阵			
identity matrix	单位矩阵			

Recall

In AS Maths you solved quadratics and worked with their coefficients. Bring this with you:

- a quadratic $ax^2 + bx + c$ has coefficients a, b, c;
- you solved $ax^2 + bx + c = 0$ by factorising.

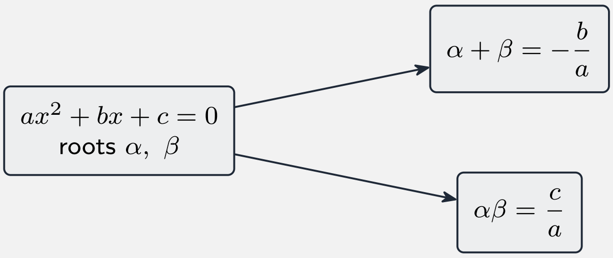
Further Pure 1 links a polynomial's **roots** to its coefficients, and adds a new object — the **matrix**.

New ideas

1 Roots of a quadratic

For $ax^2 + bx + c = 0$ with **roots** 根 α and β, the **coefficients** 系数 give the sum and product directly:

$$\alpha + \beta = -\frac{b}{a}, \qquad \alpha\beta = \frac{c}{a}.$$



Worked example. For $x^2 - 5x + 6 = 0$: $\alpha + \beta = 5$ and $\alpha\beta = 6$ (the roots are 2 and 3).

Micro-check. For $x^2 - 7x + 10 = 0$, write down $\alpha + \beta$ and $\alpha\beta$. [2]

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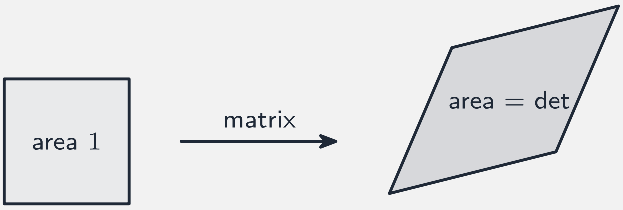
2 Matrices

A **matrix** 矩阵 is a block of numbers. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, the **determinant** 行列式 is $ad - bc$. When $ad - bc \neq 0$, the **inverse matrix** 逆矩阵 is

$$\frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

The **identity matrix** 单位矩阵 I leaves any matrix unchanged. A 2×2 matrix maps the unit square to a parallelogram whose area is the determinant.



Worked example. The determinant of $\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ is $3 \times 4 - 1 \times 2 = 10$.

Micro-check. Find the determinant of $\begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}$. [1]

Watch out: the sum of the roots is $-\dfrac{b}{a}$ —keep the minus sign. For the inverse, swap the diagonal entries a and d, negate b and c, then divide by the determinant $ad - bc$ (which must not be 0).

Practice

1.1 For $x^2 - 8x + 15 = 0$ with roots α, β, write down (a) $\alpha + \beta$ and (b) $\alpha\beta$. [2]

1.2 A quadratic $2x^2 + 6x + 4 = 0$ has roots α, β. Find $\alpha + \beta$ and $\alpha\beta$. [2]

1.3 Find the determinant of $\begin{pmatrix} 4 & 3 \\ 2 & 5 \end{pmatrix}$. [2]

1.4 Find the inverse of $\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$. [2]

1.5 State what the **identity matrix** I does when it multiplies another matrix. [1]

1.6 Challenge. The equation $x^2 - 6x + 8 = 0$ has roots α and β. Without solving it, find $\alpha^2 + \beta^2$. [2]