

## Solutions

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Each answer shows the steps, then the **result**.

### 2.1

- $G(t) = \frac{1}{3}(t + t^2 + t^3)$

### 2.2

- $E(X) = G'(1)$

### 3.1 C

- $G(1) = \sum P(X = x) = 1$

### 3.2

(a)

- $G(t) = \frac{1}{4}(1 + 2t + t^2) = \frac{1}{4} + \frac{1}{2}t + \frac{1}{4}t^2$ , so  $P(X = 0) = \frac{1}{4}$ ,  $P(X = 1) = \frac{1}{2}$ ,  $P(X = 2) = \frac{1}{4}$

(b)

- $G'(t) = \frac{1}{2} + \frac{1}{2}t \Rightarrow E(X) = G'(1) = 1$ ;  $G''(t) = \frac{1}{2} \Rightarrow \text{Var}(X) = \frac{1}{2} + 1 - 1^2 = \frac{1}{2}$

### 3.3

- $G_{X+Y}(t) = G_X(t)G_Y(t) = \frac{1}{2}(1+t) \cdot \frac{1}{3}(1+t+t^2) = \frac{1}{6}(1+t)(1+t+t^2)$ ;  $P(X+Y=0)$  is the constant term  $= \frac{1}{6}$