

Solutions

Each answer shows the steps, then the **result**.

2.1

- $a = v \frac{dv}{dx}$

2.2

- $\int v \, dv = \int 2x \, dx \Rightarrow \frac{1}{2}v^2 = x^2 + C; v = 0 \text{ at } x = 0 \Rightarrow C = 0, \text{ so } v^2 = 2x^2$

3.1 B

- Use $a = v \frac{dv}{dx}$ for a position-dependent force

3.2

(a)

- $2v \frac{dv}{dx} = 6x$, i.e. $v \frac{dv}{dx} = 3x$

(b)

- $\int v \, dv = \int 3x \, dx \Rightarrow \frac{1}{2}v^2 = \frac{3}{2}x^2 + C; \text{ at } x = 0, v = 2 \Rightarrow C = 2; \text{ at } x = 2: \frac{1}{2}v^2 = 6 + 2 = 8 \Rightarrow v^2 = 16; v = 4 \text{ m s}^{-1}$

3.3

- $v = \int (4 - 2t) \, dt = 4t - t^2 + C; v = 0 \text{ at } t = 0 \Rightarrow C = 0; \text{ at } t = 3: v = 12 - 9 = 3 \text{ m s}^{-1};$
the force has reversed (since $4 - 2t < 0$ for $t > 2$), so the particle is slowing after $t = 2$