Solutions

Each answer shows the steps, then the **result**.

2.1

- $\cos 4\theta + i \sin 4\theta$

2.2

- $1,\ \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3},\ \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$

3.1 A

- $(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta$

3.2

(a)

- imaginary part of $(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$: $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$; $= 3(1-\sin^2\theta)\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta$

(b)

- $4\sin^3\theta - 3\sin\theta = -\sin 3\theta$, so the equation is $-\sin 3\theta + \frac{1}{2} = 0 \Rightarrow \sin 3\theta = \frac{1}{2}$; for $0 < 3\theta < \frac{3\pi}{2}$, $3\theta = \frac{\pi}{6}$ or $\frac{5\pi}{6}$; $\theta = \frac{\pi}{18}$ or $\frac{5\pi}{18}$

3.3

- $z^4 = 16 \Rightarrow z = 2e^{ik\pi/2}$, $k = 0, 1, 2, 3$; $z = 2,\ 2i,\ -2,\ -2i$