

2.4 Integration (standard forms, reduction formulae)

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS completing the square 配方法 • reduction formula 递推公式
• further integration 积分 • arc length 弧长

Objective

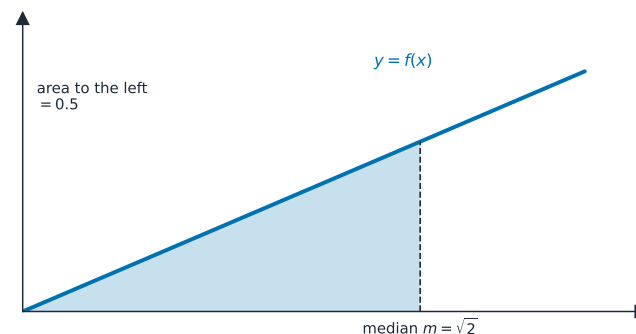
Build the skills to answer exam questions on **further integration** 积分 — the standard inverse-function integrals, **reduction formulae** 递推公式, and **arc length** 弧长.

You must be able to:

- use $\int \frac{1}{a^2+x^2} dx$ and $\int \frac{1}{\sqrt{a^2-x^2}} dx$ (and completing the square)
- derive and use a reduction formula
- find an arc length

1 Worked examples

■ Standard integrals & reduction



$$\text{median } m \text{ solves } F(m) = \int_{-\infty}^m f(x) dx = \frac{1}{2}$$

An integral is the area under a curve; the inverse-function forms handle $\frac{1}{a^2+x^2}$ and $\frac{1}{\sqrt{a^2-x^2}}$

$$\int \frac{1}{\sqrt{4-x^2}} dx = \sin^{-1} \frac{x}{2} + C.$$

Reduction: $I_n = \int_0^{\pi/2} \sin^n x dx$ satisfies $I_n = \frac{n-1}{n} I_{n-2}$.

2 Practice

2.1 Find $\int \frac{1}{9+x^2} dx$. [2]

2.2 Find $\int \frac{1}{\sqrt{25-x^2}} dx$. [2]

3 Exam-style questions

3.1 $\int_0^2 \frac{1}{4+x^2} dx$ equals: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A $\frac{\pi}{8}$
B $\frac{\pi}{4}$
C $\frac{\pi}{2}$
D π

3.2 Let $I_n = \int_0^{\pi/2} \sin^n x \, dx$.

(a) Using integration by parts, show that $I_n = \frac{n-1}{n} I_{n-2}$ for $n \geq 2$. [4]

(b) Hence evaluate I_4 . [3]

3.3 Find $\int \frac{1}{x^2 - 4x + 13} dx$ by completing the square. [4]
