Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{1}{3}\tan^{-1}\dfrac{x}{3}+C$

2.2

- $\sin^{-1}\dfrac{x}{5}+C$

3.1 A

- $\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2=\frac{1}{2}\tan^{-1}1=\frac{1}{2}\cdot\frac{\pi}{4}=\dfrac{\pi}{8}$

3.2

(a)

- $I_n=\int_0^{\pi/2}\sin^{n-1}x\sin x\,dx$; by parts ($u=\sin^{n-1}x$, $dv=\sin x\,dx$): $I_n=[-\sin^{n-1}x\cos x]_0^{\pi/2}+(n-1)\int_0^{\pi/2}\sin^{n-2}x\cos^2x\,dx$; the boundary term is 0; $\cos^2x=1-\sin^2x$ gives $I_n=(n-1)(I_{n-2}-I_n)$; so $nI_n=(n-1)I_{n-2}$, i.e. $I_n=\dfrac{n-1}{n}I_{n-2}$

(b)

- $I_0=\dfrac{\pi}{2}$; $I_2=\frac{1}{2}I_0=\dfrac{\pi}{4}$; $I_4=\frac{3}{4}I_2=\dfrac{3\pi}{16}$

3.3

- $x^2-4x+13=(x-2)^2+9$; $\displaystyle\int\frac{1}{(x-2)^2+9}\,dx=\frac{1}{3}\tan^{-1}\frac{x-2}{3}+C$

3.4

(a)

- $\displaystyle\int_0^{\ln 2}\cosh x\,dx=\Big[\sinh x\Big]_0^{\ln 2}=\sinh(\ln 2)$
- $=\frac{1}{2}\left(2-\frac{1}{2}\right)=\frac{3}{4}$

(b)

- $\sinh^{-1}\!\left(\dfrac{x}{3}\right)+c$, or $\ln\!\left(x+\sqrt{x^2+9}\right)+c$

3.5

- $\dfrac{dy}{dx}=x^{1/2}$, so $s=\displaystyle\int_0^3\sqrt{1+x}\,dx$
- $=\left[\frac{2}{3}(1+x)^{3/2}\right]_0^3=\frac{2}{3}(8-1)$
- $s=\mathbf{\dfrac{14}{3}}$

3.6

- width 0.25; $\dfrac{1}{x}$ is decreasing, so right-hand heights give the lower bound and left-hand heights the upper bound
- lower $=0.25\left(\frac{1}{1.25}+\frac{1}{1.5}+\frac{1}{1.75}+\frac{1}{2}\right)=\mathbf{0.635}$
- upper $=0.25\left(1+\frac{1}{1.25}+\frac{1}{1.5}+\frac{1}{1.75}\right)=\mathbf{0.760}$