Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{1}{3}\tan^{-1}\dfrac{x}{3} + C$

2.2

- $\sin^{-1}\dfrac{x}{5} + C$

3.1 A

- $\left[\frac{1}{2}\tan^{-1}\frac{x}{2}\right]_0^2 = \frac{1}{2}\tan^{-1}1 = \frac{1}{2}\cdot\frac{\pi}{4} = \dfrac{\pi}{8}$

3.2

(a)

- $I_n = \int_0^{\pi/2}\sin^{n-1}x\sin x\,dx$; by parts ($u = \sin^{n-1}x$, $dv = \sin x\,dx$): $I_n = [-\sin^{n-1}x\cos x]_0^{\pi/2} + (n-1)\int_0^{\pi/2}\sin^{n-2}x\cos^2 x\,dx$; the boundary term is 0; $\cos^2 x = 1 - \sin^2 x$ gives $I_n = (n-1)(I_{n-2} - I_n)$; so $nI_n = (n-1)I_{n-2}$, i.e. $I_n = \dfrac{n-1}{n}I_{n-2}$

(b)

- $I_0 = \dfrac{\pi}{2}$; $I_2 = \frac{1}{2}I_0 = \dfrac{\pi}{4}$; $I_4 = \frac{3}{4}I_2 = \dfrac{3\pi}{16}$

3.3

- $x^2 - 4x + 13 = (x-2)^2 + 9$; $\displaystyle\int \frac{1}{(x-2)^2+9}\,dx = \frac{1}{3}\tan^{-1}\frac{x-2}{3} + C$