Solutions

Each answer shows the steps, then the **result**.

2.1

- $\dfrac{dy}{dx} = 3\sinh(3x)$

2.2

- $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{1-4x^2}}$

2.3

- $\dfrac{d}{dx}\cosh 4x = 4\sinh 4x$; $\dfrac{d}{dx}\arctan 2x = \dfrac{2}{1+4x^2}$

2.4

- $x = 3\cosh u$; because $\cosh^2 u - 1 = \sinh^2 u$ removes the root when $a = 3$

2.5

- $\cosh^2 x = \frac{1}{4}(e^x + e^{-x})^2 = \frac{1}{4}(e^{2x} + 2 + e^{-2x})$; $\sinh^2 x = \frac{1}{4}(e^{2x} - 2 + e^{-2x})$; subtracting, $\frac{1}{4}(4) = 1$

3.1 A

- $\dfrac{d}{dx}\tanh x = \operatorname{sech}^2 x$

3.2

- $f(0) = 0$; $f'(x) = \dfrac{1}{1+x}$, $f'(0) = 1$; $f''(x) = -\dfrac{1}{(1+x)^2}$, $f''(0) = -1$; $f'''(0) = 2$; $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \cdots$

3.3

- $\dfrac{dx}{dt} = 2t$, $\dfrac{dy}{dt} = 3t^2$, so $\dfrac{dy}{dx} = \dfrac{3t^2}{2t} = \dfrac{3t}{2}$; $\dfrac{d^2y}{dx^2} = \dfrac{d}{dt}\left(\dfrac{3t}{2}\right) \div \dfrac{dx}{dt} = \dfrac{3/2}{2t} = \dfrac{3}{4t}$

3.4

(a)

- $\dfrac{dy}{dx} = \dfrac{2}{\sqrt{(2x)^2-1}} = \dfrac{2}{\sqrt{4x^2-1}}$

(b)

- The integrand is half the derivative in

(a)

-, so the integral is $\frac{1}{2}\operatorname{arcosh}(2x) + C$

(c)

- $x = 3\sinh u$ so $\dfrac{dx}{du} = 3\cosh u$; $9 + x^2 = 9(1+\sinh^2 u) = 9\cosh^2 u$, so $\sqrt{9+x^2} = 3\cosh u$; the integral becomes $\displaystyle\int \frac{3\cosh u}{3\cosh u}\,du = \int 1\,du = u + C$; and $u = \operatorname{arsinh}(x/3)$

(d)

- $\dfrac{d}{dx}(x\arctan x) = \arctan x + \dfrac{x}{1+x^2}$; integrating both sides from 0 to 1, $[x\arctan x]_0^1 = \displaystyle\int_0^1 \arctan x\,dx + \left[\tfrac{1}{2}\ln(1+x^2)\right]_0^1$; so $\dfrac{\pi}{4} = I + \frac{1}{2}\ln 2$; $I = \dfrac{\pi}{4} - \frac{1}{2}\ln 2$