

Solutions

Each answer shows the steps, then the **result**.

2.1

- $\frac{dy}{dx} = 3 \sinh(3x)$

2.2

- $\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$

2.3

- $\frac{d}{dx} \cosh 4x = 4 \sinh 4x; \frac{d}{dx} \arctan 2x = \frac{2}{1+4x^2}$

2.4

- $x = 3 \cosh u$; because $\cosh^2 u - 1 = \sinh^2 u$ removes the root when $a = 3$

2.5

- $\cosh^2 x = \frac{1}{4}(e^x + e^{-x})^2 = \frac{1}{4}(e^{2x} + 2 + e^{-2x}); \sinh^2 x = \frac{1}{4}(e^{2x} - 2 + e^{-2x});$ subtracting,
 $\frac{1}{4}(4) = 1$

3.1 A

- $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$

3.2

- $f(0) = 0; f'(x) = \frac{1}{1+x}, f'(0) = 1; f''(x) = -\frac{1}{(1+x)^2}, f''(0) = -1; f'''(0) = 2;$
 $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$

3.3

- $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2, \text{ so } \frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}; \frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{3t}{2} \right) \div \frac{dx}{dt} = \frac{3/2}{2t} = \frac{3}{4t}$

3.4

(a)

- $\frac{dy}{dx} = \frac{2}{\sqrt{(2x)^2 - 1}} = \frac{2}{\sqrt{4x^2 - 1}}$

(b)

- The integrand is half the derivative in

(a)

-, so the integral is $\frac{1}{2} \operatorname{arcosh}(2x) + C$

(c)

- $x = 3 \sinh u$ so $\frac{dx}{du} = 3 \cosh u$; $9 + x^2 = 9(1 + \sinh^2 u) = 9 \cosh^2 u$, so $\sqrt{9 + x^2} = 3 \cosh u$; the integral becomes $\int \frac{3 \cosh u}{3 \cosh u} du = \int 1 du = u + C$; and $u = \operatorname{arsinh}(x/3)$

(d)

- $\frac{d}{dx}(x \arctan x) = \arctan x + \frac{x}{1 + x^2}$; integrating both sides from 0 to 1, $[x \arctan x]_0^1 = \int_0^1 \arctan x dx + [\frac{1}{2} \ln(1 + x^2)]_0^1$; so $\frac{\pi}{4} = I + \frac{1}{2} \ln 2$; $I = \frac{\pi}{4} - \frac{1}{2} \ln 2$