

2.1 Hyperbolic functions

Name: _____ Class: _____ Date: _____

Total: 13 marks**KEY WORDS** hyperbolic functions 双曲函数 • logarithmic form 对数形式

Objective

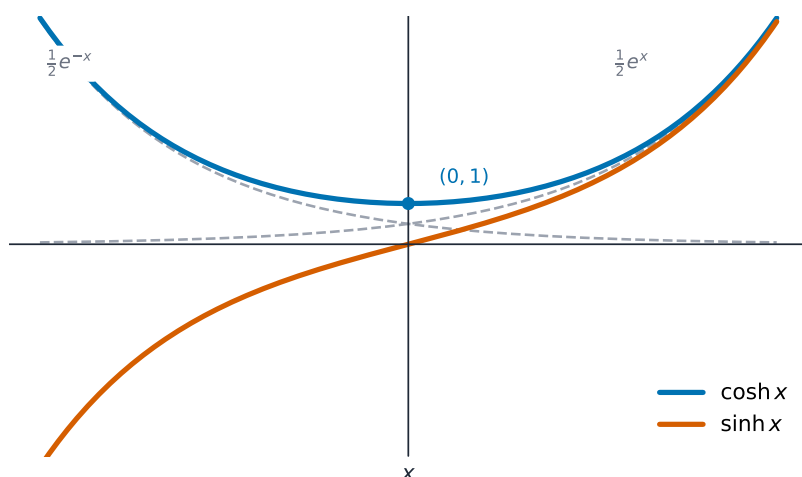
Build the skills to answer exam questions on **hyperbolic functions** 双曲函数 — the definitions, identities, and the **logarithmic form** 对数形式 of the inverses.

You must be able to:

- use $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$
- use $\cosh^2 x - \sinh^2 x = 1$ and related identities
- use $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$ and solve equations

1 Worked examples

■ Identity & solving



$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \sinh x = \frac{e^x - e^{-x}}{2} \quad \cosh^2 x - \sinh^2 x = 1$$

cosh is the average of e^x and e^{-x} ; sinh is half their difference

$$\cosh^2 x - \sinh^2 x = \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{4} = \frac{4}{4} = 1.$$

Solve $\cosh x = 2$: $\frac{e^x + e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0 \Rightarrow e^x = 2 \pm \sqrt{3}, x = \ln(2 \pm \sqrt{3}).$

2 Practice

2.1 Write down the value of $\cosh 0$.

[1]

2.2 Evaluate $\sinh^{-1} 0$ using the logarithmic form.

[1]

3 Exam-style questions

3.1 The identity satisfied by hyperbolic functions is:

[1]

A	B
C	D

A $\cosh^2 x + \sinh^2 x = 1$

B $\cosh^2 x - \sinh^2 x = 1$

C $\sinh^2 x - \cosh^2 x = 1$

D $\cosh^2 x - \sinh^2 x = -1$

3.2 Solve the equation $\cosh x = 3$, giving exact answers. [4]

3.3 (a) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$. [4]

(b) Hence evaluate $\sinh^{-1}(\frac{3}{4})$ as a single logarithm. [2]
