Solutions

Each answer shows the steps, then the **result**.

2.1

- base case (true for $n = 1$); inductive step (assume $n = k$, prove $n = k + 1$)

2.2

- LHS $= 1$, RHS $= \frac{1}{6}(1)(2)(3) = 1$ ✓

3.1 C

- The inductive step proves the result for $n = k + 1$

3.2

- base $n = 1$: LHS $= 2$, RHS $= \frac{1}{3}(1)(2)(3) = 2$ ✓; assume $\sum_1^k r(r+1) = \frac{1}{3}k(k+1)(k+2)$; $\sum_1^{k+1} = \frac{1}{3}k(k+1)(k+2)+(k+1)(k+2)$; $= (k+1)(k+2)\left(\frac{k}{3} + 1\right) = \frac{1}{3}(k+1)(k+2)(k+3)$; this is the formula at $n = k + 1$, so true for all n

3.3

- base $n = 1$: $3^2 - 1 = 8$, divisible by 8 ✓; assume $3^{2k} - 1 = 8m$ for some integer m, so $3^{2k} = 8m + 1$; then $3^{2(k+1)} - 1 = 3^2 \cdot 3^{2k} - 1 = 9(8m + 1) - 1 = 72m + 8$; $= 8(9m + 1)$, and $9m + 1$ is an integer; so the result holds at $n = k + 1$ whenever it holds at $n = k$, and by induction for all positive n