

1.6 Vectors

Name: _____ Class: _____ Date: _____

Total: 18 marks

KEY WORDS plane 平面 • vectors 向量 • vector product 向量积

Objective

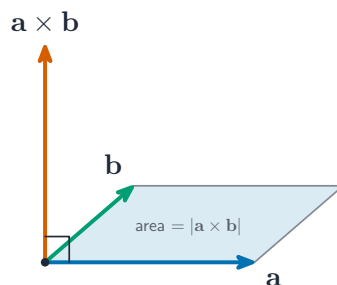
Build the skills to answer exam questions on **vectors** 向量 in 3-D — the **vector product** 向量积, equations of a **plane** 平面, and distances/angles between lines and planes.

You must be able to:

- compute $\mathbf{a} \times \mathbf{b}$ and use it for a normal/area
- write a plane as $\mathbf{r} \cdot \mathbf{n} = p$ or $ax + by + cz = d$
- find the angle between, and distance from, lines and planes

1 Worked examples

■ Vector product & plane



$\mathbf{a} \times \mathbf{b}$ is perpendicular to both; its length is the area of the parallelogram

$\mathbf{a} = \mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{j} + \mathbf{k}$: $\mathbf{a} \times \mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$.

Plane through A with normal $\mathbf{n}$: $\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$.

2 Practice

2.1 Find $\mathbf{a} \times \mathbf{b}$ for $\mathbf{a} = 2\mathbf{i} + \mathbf{j}$, $\mathbf{b} = \mathbf{i} + \mathbf{k}$. [3]

2.2 Write the plane through $(1, 2, 3)$ with normal $\mathbf{n} = (1, 1, 1)$ in the form $ax + by + cz = d$. [2]

3 Exam-style questions

3.1 $\mathbf{a} \times \mathbf{b}$ is a vector that is: [1]

- | | |
|---|---|
| A | B |
| C | D |
- A parallel to $\mathbf{a}$
 B perpendicular to both $\mathbf{a}$ and $\mathbf{b}$
 C equal to $\mathbf{a} \cdot \mathbf{b}$
 D of length $|\mathbf{a}||\mathbf{b}|\cos\theta$

3.2 Points $P(1, 0, 2)$, $Q(3, 1, 1)$ and $R(2, -2, 4)$ are given.

(a) Find $\overrightarrow{PQ} \times \overrightarrow{PR}$, and hence the area of triangle PQR . [4]

(b) Find the equation of the plane PQR , and the perpendicular distance from the origin to it. [4]

3.3 Find the acute angle between the planes $x + 2y + 2z = 5$ and $2x - y + 2z = 1$. [4]
