Solutions

Each answer shows the steps, then the **result**.

2.1

- $\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \mathbf{i}(1) - \mathbf{j}(2) + \mathbf{k}(-1) = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$

2.2

- $x + y + z = 1 + 2 + 3 = 6$

3.1 B

- The vector product is perpendicular to both vectors

3.2

(a)

- $\overrightarrow{PQ} = (2, 1, -1)$, $\overrightarrow{PR} = (1, -2, 2)$; cross $= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & -1 \\ 1 & -2 & 2 \end{vmatrix} = \mathbf{i}(2-2) - \mathbf{j}(4+1) + \mathbf{k}(-4-1) = (0, -5, -5)$; the triangle is half the parallelogram, so its area is $\frac{1}{2}\,|(0, -5, -5)| = \frac{1}{2} \cdot 5\sqrt{2} = \frac{5\sqrt{2}}{2}$

(b)

- any parallel normal will do, so take $(0, 1, 1)$; through $P(1, 0, 2)$ this gives $y + z = 2$. Distance from the origin $= \dfrac{|0 + 0 - 2|}{\sqrt{0^2 + 1^2 + 1^2}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

3.3

- normals $(1, 2, 2)$ and $(2, -1, 2)$; dot $= 2 - 2 + 4 = 4$; $|\mathbf{n}_1| = 3$, $|\mathbf{n}_2| = 3$; $\cos\theta = \dfrac{4}{9} \Rightarrow \theta = 63.6°$