A-LEVEL FURTHER MATHEMATICS • EXERCISE SHEET

1.5 Polar coordinates

Name: _____ Class: _____ Date: _____ **Total: 16 marks**

KEY WORDS polar coordinates 极坐标 • polar curves 极坐标曲线 • cartesian coordinates 直角坐标

Objective

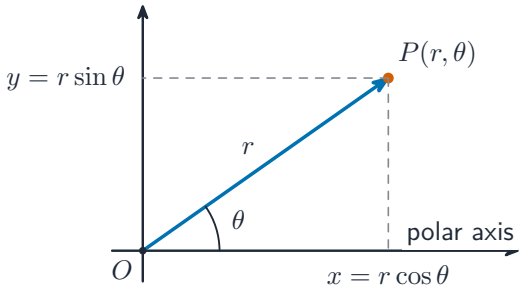
Build the skills to answer exam questions on **polar coordinates** 极坐标 — converting to/from Cartesian, sketching **polar curves** 极坐标曲线, and the **sector area** $\frac{1}{2}\int r^2\,d\theta$.

You must be able to:

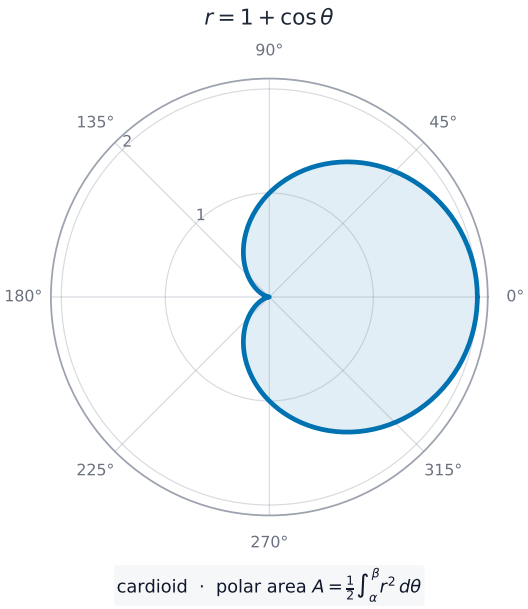
- convert between (r,θ) and (x,y)
- sketch a simple polar curve (e.g. a cardioid)
- find an area with $\frac{1}{2}\int r^2\,d\theta$

1 Worked examples

■ Conversion & area



$$x = r\cos\theta,\ y = r\sin\theta,\ r^2 = x^2 + y^2$$



Polar curves are traced as the angle varies

Convert $r = 4\cos\theta$**:** $\times r \to r^2 = 4r\cos\theta$, so $x^2 + y^2 = 4x$, i.e. $(x-2)^2 + y^2 = 4$ (circle).

Area: $A = \frac{1}{2}\displaystyle\int_\alpha^\beta r^2\,d\theta$.

2 Practice

2.1 Convert the point $(r, \theta) = \left(2, \frac{\pi}{3}\right)$ to Cartesian coordinates. [2]

2.2 Write down the formula for the area of a polar sector. [1]

3 Exam-style questions

3.1 The polar equation $r = 2\sin\theta$ represents: [1]

A B
C D

A a line
B a circle
C a parabola
D a spiral

3.2 A curve has polar equation $r = 3 + 2\cos\theta$ for $0 \leq \theta < 2\pi$ (a convex limaçon).

(a) Write down the greatest and least values of r, and state the value of θ at which each occurs. Explain why the curve never passes through the pole. [3]

(b) Show that the area enclosed by the curve is 11π. [5]

3.3 Convert $r = \dfrac{2}{1 + \cos\theta}$ to Cartesian form. [4]